

**Four Year Degree Program of  
Mathematics**

**BS (Mathematics)**

**DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF KARACHI**

### **Program Overview:**

The four year degree program of Bachelor of Science (BS) in Mathematics is designed to provide students with a robust foundation in mathematical theories, methodologies, and applications. This program is structured to cultivate a deep understanding of both pure and applied mathematics, equipping students with the skills necessary to pursue a variety of careers or advanced studies in mathematics and related fields.

### **Program Objectives**

The BS Mathematics program will possess a comprehensive understanding of core mathematical principles and theories, enabling students to applying mathematical techniques and critical thinking skills to solve complex, real-world problems across various domains such as science, medical engineering, economics, and others. The BS program aims to achieve the following objectives:

- ❖ **Develop Fundamental Knowledge:** Provide students with a solid foundation in mathematical theories, principles, and methods, ensuring they have a comprehensive understanding of both pure and applied mathematics.
- ❖ **Enhance Analytical Skills:** Cultivate students' abilities to think critically and analytically, enabling them to solve complex mathematical problems and make informed decisions based on quantitative data.
- ❖ **Foster Research and Innovation:** Encourage and support students in conducting original research, fostering an environment of curiosity and innovation where new mathematical ideas and solutions can flourish.
- ❖ **Promote Interdisciplinary Applications:** Demonstrate the relevance and applicability of mathematics across various disciplines, such as engineering, technology, finance, and social sciences, preparing students for diverse career opportunities.
- ❖ **Prepare for Professional Success:** Prepare students for successful careers in academia, industry, government, and other sectors by providing them with the mathematical knowledge and skills necessary to excel in their chosen fields.
- ❖ **Engage with the Community:** Encourage students to engage with the broader community through outreach, service, and collaborative projects, demonstrating the importance and impact of mathematics in societal development and problem-solving.

**DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF KARACHI**

**Total Credit Hours: 140**

		Sr. No.	Course No.	Course Title	Credit Hours	Course Type
		<b>YEAR - 1</b>	<b>SEMESTER - I</b>	1	300.1 (E)	English - I
2	300.1 (I. S)			Islamic Studies OR Ethics (Non-Muslim)	2+0	Gen Ed
3	300.1 (Civ/Com)			Civics and Community Engagement	2+0	Gen Ed
4	300.1 (N. Sc)			Everyday Science, Astronomy, Physics, Chemistry, Earth Science or Biology	3+0	Gen Ed
5				Minor-I	3+0	Int Disc
6				Minor-II	3+0	Int Disc
7	M301			Algebra	3+0	Major
				Total Credit Hours	18	
<b>SEMESTER - II</b>	1		300.2	Ideology & Constitution of Pakistan/Pakistan Studies	2+0/2+0	Gen Ed
	2		300.2	Functional English	3+0	Gen Ed
	3		300.2 (Entr)	Entrepreneurship	2+0	Gen Ed
	4		300.2 (Soc. Sc)	Intro to Soc. Sc., Urdu, Economics, History, Geography or Psychology.	2+0	Gen Ed
	5			Minor-I	3+0	Int Disc
	6			Minor-II	3+0	Int Disc
	7	M302	Calculus	3+0	Major	
			Total Credit Hours	20		

		Sr. No.	Course No.	Course Title	Credit Hours	Course Type
		<b>YEAR - 2</b>	<b>SEMESTER-III</b>	1	400.1 (Q. Reas)	Quantitative Reasoning -I
2	400.1 (E. Writ)			Expository Writing	3+0	Gen Ed
3				Minor-I	3+0	Int Disc
4				Minor-II	3+0	Int Disc
5	M401			Multivariable Calculus and Geometry	3+0	Major
6	M403			Discrete Mathematics & Graph Theory	3+0	Major
				Total Credit Hours	18	
<b>SEMESTER-IV</b>	1		400.2 (Q. Reas)	Quantitative Reasoning-II	3+0	Gen Ed
	2		400.2 (ICT)	Application of Inf. & Comm. Technologies	3+0	Gen Ed
	3			Minor-I	2+1	Sub
	4			Minor-II	3+0	Sub
	5		M402	Mechanics	3+0	Major
	6		M404	Linear Algebra	3+0	Major
				Total Credit Hours	18	

YEAR - 3	SEMESTER-V	Sr. No.	Course No.	Course Title	Credit Hours	Course Type	
		1	M501	Real Analysis	3+0	Major	
		2	M503	Abstract Algebra	3+0	Major	
		3	M505	Numerical Analysis-I	2+1	Major	
		4	M507	Differential Geometry	3+0	Major	
		5	M509	Methods of Mathematical Physics	3+0	Major	
					Total Credit Hours	15	
	SEMESTER-VI	1	M502	Complex Analysis	3+0	Major	
		2	M504	Topology	3+0	Major	
		3	M506	Numerical Analysis-II	2+1	Major	
		4	M508	Functional Analysis-I	3+0	Major	
		5	M510	Integral Transforms and Special Functions	3+0	Major	
						Total Credit Hours	15

### Pure Mathematics

YEAR - 4	SEMESTER - VII	Sr. No.	Course No.	Course Title	Credit Hours	Course Type	
		1	M601	Theory of Rings & Modules	3+0	Major / (Opt)	
		2	M603	Measure Theory-I	3+0	Major / (Opt)	
		3	M605	Functional Analysis-II	3+0	Major / (Opt)	
		4		Optional-I		Major / (Opt)	
		5		Optional-II		Major / (Opt)	
		6	M600.1	Field Experience / Internship	0+3	Compulsory	
					Total Credit Hours	18	
	SEMESTER - VIII	1	M602	Galois Theory and its Applications	3+0	Major / (Opt)	
		2	M604	Measure Theory-II	3+0	Major / (Opt)	
		3	M606	Algebraic Topology	3+0	Major / (Opt)	
		4		Optional-I	3+0	Major / (Opt)	
		5		Optional-II	3+0	Major / (Opt)	
		6	M600.2	Capstone Project	0+3	Compulsory	
				Total Credit Hours	18		

### Applied Mathematics

YEAR - 4	SEMESTER - VII	Sr. No.	Course No.	Course Title	Credit Hours	Course Type	
		1	M651	Classical Mechanics-I	3+0	Major / (Opt)	
		2	M655	Fluid Dynamics-I	3+0	Major / (Opt)	
		3	M653	Tensor Analysis	3+0	Major / (Opt)	
		4		Optional-I		Major / (Opt)	
		5		Optional-II		Major / (Opt)	
		6	M600.1	Field Experience / Internship	0+3	Compulsory	
					Total Credit Hours	18	
	SEMESTER - VIII	1	M652	Classical Mechanics-II	3+0	Major / (Opt)	
		2	M656	Fluid Dynamics-II	3+0	Major / (Opt)	
		3	M654	Partial Differential Equations	3+0	Major / (Opt)	
		4		Optional-I		Major / (Opt)	
		5		Optional-II		Major / (Opt)	
		6	M600.2	Capstone Project	0+3	Compulsory	
				Total Credit Hours	18		

## Optional Subjects

Optional Courses for Seventh Semester		Credit Hours
1.	M – 615 SUMMABILITY THEORY-I	3 + 0 (only for Pure Maths)
2.	M – 647 PROJECTIVE GEOMETRY-I	3 + 0 (only for Pure Maths)
3.	M – 661 ELECTROMAGNETICS-I	3 + 0 (only for Appl. Maths)
4.	M – 671 RELATIVITY-I	3 + 0 (only for Appl. Maths)
5.	M – 685 ASTRONOMY-I	3 + 0 (only for Appl. Maths)
6.	M – 631 NUMERICAL METHODS	3 + 0 (Both Pure & Applied)
7.	*M – 633 MATHEMATICAL STATISTICS-I	3 + 0 (Both Pure & Applied)
8.	M – 637 APPLIED ALGEBRA-I	2 + 1 (Both Pure & Applied)
9.	M – 645 OPERATIONS RESEARCH-I	3 + 0 (Both Pure & Applied)
10.	M – 649 STOCHASTIC PROCESSES	3 + 0 (Both Pure & Applied)
11.	M – 657 NONLINEAR SYSTEMS-I	3 + 0 (Both Pure & Applied)
12.	M – 687 ADVANCED STATISTICS-I	3 + 0 (Both Pure & Applied)
13.	M – 689 MATHEMATICAL FINANCE-I	3 + 0 (Both Pure & Applied)
14.	M – 691 BIOLOGICAL MODELING	3 + 0 (Both Pure & Applied)
15.	M – 693 OPTIMIZATION THEORY	3 + 0 (Both Pure & Applied)
16.	M – 695 COMPUTATIONAL NUMBER THEORY	3 + 0 (Both Pure & Applied)
17.	M – 697 ACTUARIAL MATHEMATICS-I	3 + 0 (Both Pure & Applied)
18.	M – 663 INTRODUCTION TO COMPUTER PROGRAMMING	2 + 1 (Both Pure & Applied)

Optional Courses for Eight Semester		Credit Hours
1.	M – 616 SUMMABILITY THEORY-II	3 + 0 (only for Pure Maths)
2.	M – 648 PROJECTIVE GEOMETRY-II	3 + 0 (only for Pure Maths)
3.	M – 662 ELECTROMAGNETICS-II	3 + 0 (only for Appl. Maths)
4.	M – 672 RELATIVITY-II	3 + 0 (only for Appl. Maths)
5.	M – 686 ASTRONOMY-II	3 + 0 (only for Appl. Maths)
6.	M – 632 INTEGRAL EQUATIONS	3 + 0 (Both Pure & Applied)
7.	*M – 634 MATHEMATICAL STATISTICS-II	3 + 0 (Both Pure & Applied)
8.	M – 638 APPLIED ALGEBRA-II	2 + 1 (Both Pure & Applied)
9.	M – 646 OPERATIONS RESEARCH-II	3 + 0 (Both Pure & Applied)
10.	M – 650 RENEWAL PROCESSES & THEORY OF QUEUES	3 + 0 (Both Pure & Applied)
11.	M – 658 NONLINEAR SYSTEMS-II	3 + 0 (Both Pure & Applied)
12.	M – 688 ADVANCED STATISTICS-II	3 + 0 (Both Pure & Applied)
13.	M – 690 MATHEMATICAL FINANCE-II	3 + 0 (Both Pure & Applied)
14.	M – 692 DISCRETE MAPS	3 + 0 (Both Pure & Applied)
15.	M – 694 MACHINE LEARNING & DATA SCIENCE	2 + 1 (Both Pure & Applied)
16.	M – 696 SCIENTIFIC COMPUTING	2 + 1 (Both Pure & Applied)
17.	M – 698 ACTUARIAL MATHEMATICS-II	3 + 0 (Both Pure & Applied)
18.	M – 664 PROGRAMMING in PYTHON	2 + 1 (Both Pure & Applied)

N.B:

- 1- Courses M-301, M-302, M-401, M-402 for minor/interdisciplinary subjects.
- 2- Courses M-633, M-634 only for those students who studied BS **without** minor/interdisciplinary statistics, however M-687, M-688 may be opted by those students who studied BS **with** minor/interdisciplinary statistics.
- 3- The optional courses M-631, M-691, M-696, M-632, M-692, M-694, M-696 are independent courses.

# BS-I (Mathematics)

## First Year

SEMESTER I			SEMESTER II		
Course #	Course Title	Cr. Hr.	Course #	Course Title	Cr. Hr.
300.1 (E)	English-I	2+0	300.2	Ideology & Constitution of Pakistan/ Pakistan Studies	2+0/ 2+0
300.1 (I/S)	Islamic Studies or Ethics (Non-Muslim)	2+0	300.2	Functional English	2+0
300.1 (Civ/Com)	Civic and Community Engagement	2+0	300.2 (Entr)	Entrepreneurship	2+1
300.1 (N. Sc)	Everyday Science, Astronomy, Physics, Chemistry, Earth Science or Biology	3+0	300.2 (Soc. Sc)	Introduction to Social Science, Urdu, Economics, History, Geography or Psychology	3+0
	Minor-I	3+0		Minor -I	3+0
	Minor -II	3+0		Minor -II	3+0
M301	Algebra	3+0	M302	Calculus	3+0
	Total Credit Hours	18		Total Credit Hours	18

## *First Semester*

### M-301 Algebra ( 3 + 0 )

**Number system:** Real and complex number systems, De-Moivre's theorem with applications, exponential, trigonometric, hyperbolic, logarithmic, inverse hyperbolic and inverse circular functions.

**Group theory:** Groups and their properties, subgroups order of a group, cyclic groups, cosets, Lagrange's theorem, permutation groups, Rings, fields, vector spaces, the four fundamental subspaces; independence, basis and dimension; linear transformations and matrix space.

**Matrices:** Elementary row operations; echelon and reduced echelon forms; Inverse, rank and normal form of a matrix.

**Determinants:** Axiomatic definition of a determinant; properties and formulas of determinant and Cofactors; inverse of a matrix; volume.

**System of linear equations:** Gauss elimination and Gauss Jordan methods, Cramer's rule, consistent and inconsistent systems.

**Equations:** Solutions of cubic and biquadratic equations (Cardano and Ferrari Methods), numerical solution of equations, bisection, Newton-Raphson and Regula-Falsi methods.

**Probability:** Axioms of probability, conditional probability, Discrete and continuous random variables, Probability distributions, binomial, Poisson's and normal distributions.

### Recommended Books:

1. Jain R.K., Lyengar, S.R.K., Advanced Engineering Mathematics, Narosa Publishing House, New Delhi, 3<sup>rd</sup> edition, 2007.
2. Kreyszig E., Advanced Engineering Mathematics, John Wiley & Sons, Singapore, 9<sup>th</sup> edition, 2005.
3. Mathews J.H., Howell, R.W., Complex Analysis for Mathematics and Engineering, Jones and Bartlett Publishers, Boston, 5<sup>th</sup> edition, 2006.
4. Strang G., Introduction to Linear Algebra, 5th Edition, MIT Maths.
5. Yousuf S.M., Majeed A., Amin M., Mathematical Methods, IlmiKitab Khanna, 2016.

## *Second Semester*

### **M-302 Calculus ( 3 + 0 )**

**Differential Calculus:** Bounds, limits and continuity, Properties of continuous functions, Derivatives, Leibnitz and Rolle's theorems, Lagrange's and Cauchy's mean value theorems, generalized mean value theorem, Indeterminate forms, Taylor's and Maclaurin's series.

**Infinite Series:** Sequences, limit and bound of sequences, series, comparison test, limit comparison test, Integral test, ratio and root test, alternating series, absolute and conditional convergence.

**Integral calculus:** Anti-derivatives, Techniques of integration, Riemann integral, Properties of definite integrals, Mean value theorem, Reduction formulae, Improper integrals, Beta and Gamma integrals.

**Differential equations I:** Differential equations, formation and solution, equations of first order, Initial and boundary value problems, Various methods of solving first order differential equations, Orthogonal trajectories, Nonlinear first order equations, envelopes and singular solutions.

**Differential equations II:** Higher order differential equations with constant coefficients, Superposition of solutions, Cauchy Euler's equations, Systems of first order linear homogenous equations, nonlinear equation discussed with the help of population growth.

### **Recommended Books:**

1. Anton H., Bevens I., Davis S., Calculus, 8<sup>th</sup>edition, John Wiley & Sons, Inc. 2005
2. Apostol, Tom M. Calculus, Volume 1: One-Variable Calculus, with An Introduction to Linear Algebra, 2<sup>nd</sup> edition.
3. Hughes-Hallett, Gleason, McCallum, et al, Calculus: Multivariable, 7<sup>th</sup>edition, 2017.
4. Thomas F., Calculus, 11<sup>th</sup>edition. Addison Wesley Publishing Company, 2005
5. Yousuf S.M., Majeed A., Amin M., Mathematical Methods, IlmiKitab Khanna, 2016.
6. Talpur MNM., Calculus & Analytic Geometry , Lahore Ferozons Ltd,1982.

## Second Year

SEMESTER III			SEMESTER IV		
Course #	Course Title	Cr. Hr	Course #	Course Title	Cr. Hr
400.1 (Q.Reas)	Quantitative Reasoning-I	3+0	400.2 (Q.Reas)	Quantitative Reasoning-II	3+0
400.1 (E.Writ)	Expository Writing	3+0	400.2 (ICT)	Application of Inf. & Comm. Technologies	3+0
	Minor-I	2+1		Minor -I	2+1
	Minor -II	3+0		Minor-II	3+0
M-401	Multivariable Calculus and Geometry	3+0	M-402	Mechanics	3+0
M-403	Discrete Mathematics and Graphs Theory	3+0	M-404	Linear Algebra	3+0
	Total Credit Hours	18		Total Credit Hours	18

### *Third Semester*

#### **400.1 (Q.Reas) Quantitative Reasoning-I (3+0)**

**Numeric Reasoning:** Number system, basic arithmetic equations, units and conversions, dimensions, rates, ratios, percentage, scientific notation, computation with real number, operations of integers, exponent, square root, measurement scales.

**Algebraic and Geometric Reasoning:** Basics of geometry, line, angle, circle, polygon, area, perimeter, volume, surface etc., introduction of set, properties and operation, functions, types of functions, graphical representation of functions, relations, types of relations, simplifying algebraic expressions exponents, factorization, algebraic solutions of linear and quadratic equations.

**Logical and critical reasoning:** Logic, proposition, propositional equivalence, truth table, Conjunction, disjunction, negation, propositions, logical fallacies, tautologies and contradictions, logical equivalence, Venn diagram, components of critical thinking, observation, analysis, interpretation, reflection, evaluation, inference, scientific reasoning.

**Mathematical Modelling and Analysis:** Introduction to deterministic models, linear and non-linear functions, system of linear equations, application of derivatives, linear and exponential growth decay model.

#### **Recommended Books:**

1. Alan F. Beardon. (2005). Algebra and Geometry, 1st edition, Cambridge University Press.
2. Eric Zaslow. (2020). Quantitative Reasoning, 1st edition, Cambridge University Press.
3. Forest Jim. (2020). Introduction to Statistics: An Intuitive Guide for Analyzing Data and Unlocking Discoveries, Jim Publishing.
4. Rosen, K. H. (2018). Discrete Mathematics and its Applications, 7<sup>th</sup> edition, McGraw Hill.
5. Sevilla. A & Somers K. (2008). Quantitative Reasoning: Tools for Today's Informed Citizen, 1<sup>st</sup> edition, Wiley.
6. Bennett.J & Briggs.W. (2018). Using & Understanding Mathematics: A Quantitative Reasoning Approach, 7th edition, Pearson.
7. Frank S Budnick. (1993). Applied Mathematics for Business, Economics, and the Social Sciences, 4th edition, McGraw Hill.
8. William Fox. (2017). Mathematical Modeling for Business Analytics, 1st edition, CRC Press.



## **M-401 Multivariable Calculus and Geometry ( 3 + 0 )**

**Multivariable Calculus:** Partial derivatives, geometrical meaning, Equation of tangent plane and normal to surfaces, Chain rule, approximation with the help of differentials, Homogeneous functions, Euler's theorem, Jacobians, Evaluation of simple double and triple integrals, volume and surface areas of solids of revolution.

**Plane Curves:** Curves in Cartesian plane, parametric representation, polar coordinates, Tangents and normal, Polar equation of a conic, Pedal equations, Change of axes, General equation of second degree, Extreme values, singular points, asymptotes, curve tracing length of arc, intrinsic equation, curvature, Areas in rectangular and polar coordinates.

**Solid Geometry:** Direction cosines and direction ratios, Equations of a line, angle between two lines, distance of a point from a line, shortest distance between two lines, Equation of a plane, angle between planes, Area of a triangle and volume of tetrahedron, Spherical and cylindrical polar coordinates Surfaces, intercepts, traces, symmetry, quartic surfaces, sphere, surface of revolution, ruled surfaces.

**Differential Geometry:** Simple arcs and curves in three dimensions, their parametric representation, The arc length, the natural parameterization, Contacts (of order up to two) of curves and a Surface, Osculating plane, Frenet trihedron and Frenet formulae, curvature and torsion of surface, Surfaces in space, curvilinear coordinates, implicit equation of surface, Tangent plane, Curves on surfaces and tangent vector, Angle between curves on a surface, First and second fundamental forms on a surface.

### **Recommended Books:**

1. Anton H., Bevens I., Davis S., Calculus Early Transcendentals, 10<sup>th</sup>edition, 2015.
2. Dineen S., Multivariate calculus and geometry, Springer, 2<sup>nd</sup>edition, 2014.
3. Lax P. D., Terrell M.S., Multivariable calculus with applications, Springer, 2017.
4. Thomas F., Calculus, Addison Wesley Publishing Company, 11<sup>th</sup>edition, 2005.
5. Walschap G., Multivariable calculus and differential geometry, Walter de Gruyter GmbH & Co KG, 2015.
6. Talpur MNM., Calculus & Analytic Geometry , Lahore Ferozons Ltd,1982.

## **M-403 Discrete Mathematics and Graph Theory ( 3 + 0 )**

**Logic:** Propositions; Logical Connectives; Truth Tables, Logical Equivalence; IF-Statements, IF, IFF, Tautologies, and Contradictions, Tautologies; Quantifiers; Universes, Properties of Quantifiers, Uniqueness.

**Proofs:** Definitions, Axioms, Theorems, and Proofs, Proving Existence Statements and IF Statements, Contrapositive Proofs; IFF Proofs, Proofs by Contradiction; OR Proofs, Proof by Cases; Disproof's, Proving Universal Statements; Multiple Quantifiers.

**Boolean Algebra:** Introduction, Properties of Boolean Algebra, Boolean Functions, Switching Circuits, Logic Networks, Minimization of Boolean Expressions, Karnaugh maps

**Equivalence Relations and Partial Orders:** Reflexive, Symmetric, and Transitive Relations, Equivalence Relations, Equivalence Classes, Set Partitions, Partially Ordered Sets, Equivalence, Lattices, Lattices as algebraic, Sub lattices, Modular Lattices.

**Graph Theory:** Introduction, Incidence, adjacency, Degree, Regular, Full, Null and Bipartiate graphs.

**Sub graphs:** Disjoint, Induced, Walk, Trail and Path.

**Operations on Graphs:** Union, Intersection, Ring Sum, Wedge Union, Cartesian product, Composition, Normal and Tensor products

**Trees and Fundamental Circuits:** Cut set and cut vertices, Connectivity of a graph, Separable graphs, Trees, Pendent vertices of a tree, Distance and Centres, Rooted and Binary Trees, Binary tree in search procedure, Binary tree in coding theory, Binary Search Trees

**Planar Graph:** Planar graph, Direction of Planarity, Outer Planar graphs, Geometric dual, combinatorial dual.

**Matrix Representation:** Incidence matrix, Reduced incidence matrix, Circuit matrix, Fundamental circuit matrix, Cut set matrix, Fundamental cut set matrix, Relationship between the matrices, Adjacency matrix, Path matrix.

**Graph Coloring:** Coloring, Independent sets, Chromatic partitioning, Chromatic Polynomial, Matchings, Coverings, Switching functions, Five color problem, Four color problem.

**Directed Graphs:** Binary Relations, Relation matrix, Euler diagraph, Teleprinter's problem, A cyclic diagraph, Arborescence, Polish notation Fundamental circuits in diagraph, Incidence matrix of a digraph, Circuit matrix of a digraph, Adjacency matrix of a digraph.

### **Recommended Books:**

1. Kolman B., Busby R.C., Ross S.C., Discrete Mathematical Structures, Prentice-Hall of India, New Delhi, 5<sup>th</sup> edition, 2008.
2. Rosen K.H., Discrete Mathematics and its Application, McGraw-Hill, 8<sup>th</sup>edition, 2017.
3. Ross K.A., Wright C.R.B., Discrete Mathematics, 5<sup>th</sup>edition, Prentice Hall, New Jersey, 2003.
4. Susanna S. Epp. (2010). Discrete Mathematics with Applications, 4th edition, Cengage Learning.
5. Tucker A., Applied Combinatorics, John Wiley and Sons, Inc New York, 2002.

## *Fourth Semester*

### **400.2 (Q. Reas) Quantitative Reasoning-II (3+0)**

**Fundamental Statistical Concepts:** Population and sample Interpretation of Tabular and Graphical form of data (Grouped and ungrouped). Summarizing data; Measures of central tendency, dispersion and Quantiles.

**Combinatorial Analysis:** Rules of counting (multiplicative, permutation and combination); Venn diagram

**Basic concept of probability:** Axioms of Probability. Introduction to probability models for continuous and discrete variables; Normal and Binomial distribution with simple applications.

**Bivariate Data analysis:** Scatter plots; Pearson correlation; Simple linear regression with simple application

**Fundamental Concepts of Inference:** Basic ideas of test of significance and testing of hypothesis. Concepts of level of significance and degree of freedom.

Quantitative reasoning exercises using fundamental statistical concepts

### **Recommended Books:**

Bennett, J., & Briggs, W. (2019). Using & understanding mathematics: a quantitative reasoning approach. Pearson.

Mann, P. S. (2010). Introductory statistics. John Wiley & Sons.

Chatfield, C. (2018). Statistics for technology: a course in applied statistics. Routledge.

Lock, R. H., Lock, P. F., Morgan, K. L., Lock, E. F., & Lock, D. F. (2020). Statistics: Unlocking the power of data. John Wiley & Sons.

Peck, R., Short, T., & Olsen, C. (2020). Introduction to statistics and data analysis. Cengage Learning.

### **M-402 Mechanics ( 3 + 0 )**

**Vector Analysis:** Differentiation and integration of vectors, Scalar and vector fields, gradient, divergence and curl, Line, surface and volume integrals, Theorems of Green, Gauss and Stoke (without proofs), Applications.

**Statics:** Composition of forces, equilibrium problems, moments and couples, Centres of mass and gravity, Friction, virtual work, flexible cables, catenaries

**Dynamics:** Kinematics, Rectilinear motion with variable accelerations simple harmonic motion, Methods of dynamics, principles of energy and momentum, Motion of Projectile, Orbital motion, Moment of inertia, Motion of a rigid body, Plane impulsive motion Compound pendulum, Galilean-Newtonian principle, inertial frames, Galilean transformations.

### **Recommended Books:**

1. Ghorl, Q. K., (Ed.), Introduction to Mechanics, West Pakistan Publishing Co. 1971.
2. Haberman, Richard. Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow. SIAM, 1998
3. Kleppner D., An Introduction to Mechanics, McGraw-Hill, 2<sup>nd</sup> Edition, 2013.
4. Meirovitch L., Methods of Analytical Dynamics, McGraw Hill, New York, 1<sup>st</sup> Edition, 2007.
5. Meriam, J.L., Kraige L.G., Engineering Mechanics: dynamics, John Wiley & Sons, 2012.

## **M-404 Linear Algebra ( 3 + 0 )**

**Vector spaces:** Definition and basic properties, Subspaces, Linear independence, linear combinations and span, Basis, and dimension, change of basis, orthogonal bases and projection in  $\mathbb{R}$ .

**Linear transformations:** Definition and examples, Properties of linear transformations, Range and kernel, The rank and nullity of a matrix, The matrix representation of a linear transformation, Isomorphisms, isometries, applications

**Eigenvalues:** Eigenvalues and eigenvectors, canonical forms: a model of population growth similar matrices and diagonalization, symmetric matrices and orthogonal diagonalization, Quadratic forms, Matrix differential equations, The theorem of Cayley-Hamilton and Gershgorin circles

**Inner product spaces:** Inner products, angle and orthogonality, Gram Schmidt process, QR-decomposition.

**Positive definite matrices and applications:** Symmetric matrices and positive definiteness, complex matrices, singular value decomposition, left and right inverses, pseudo inverse, Markov matrices.

### **Recommended Books:**

1. Anton H.,Rorres C., Elementary Linear Algebra: Applications Version, 10<sup>th</sup>edition, John Wiley and sons, 2010.
2. Curtis C.W., Linear Algebra, Springer 2004.
3. FriedbergS.,Insel A., Linear Algebra, 4<sup>th</sup>edition, Pearson Education Canada, 2003.
4. Grossman S. I., Elementary Linear Algebra, 5<sup>th</sup>edition, Cengage Learning, 2004.
- 5.
6. Strang, G., Introduction to Linear Algebra, 3<sup>rd</sup>edition. Wellesley, MA: Wellesley-Cambridge Press, March2003.

## **Third Year**

SEMESTER V			SEMESTER VI		
Course #	Course Title	Cr.Hr	Course #	Course Title	Cr.Hr
M-501	Real Analysis	3+0	M-502	Complex Analysis	3+0
M-503	Abstract Algebra	3+0	M-504	Topology	3+0
M-505	Numerical Analysis-I	2+1	M-506	Numerical Analysis-II	2+1
M-507	Differential Geometry	3+0	M-508	Functional Analysis-I	3+0
M-509	Methods of Mathematical Physics	3+0	M-510	Integral Transforms and Special Functions	3+0
Total Credit Hours		15	Total Credit Hours		15

## **Fifth Semester**

### **M-501 Real Analysis ( 3 + 0 )**

Algebra of sets; partition and equivalent classes; partially ordered sets and Axiom of Choice, Canonical decomposition of functions,  $\mathbb{R}^n$ ,  $n \geq 1$ ; Euclidean metric space, Completeness, Functions Convergence of sequences and completeness, Functions of several real variables; their continuity and differentiability Implicit and Inverse Function Theorems, Jacobians and emotional dependence, Taylor's Theorems, Jacobians and Minima: Language's method of undetermined multipliers, Riemann and Riemann-Stieltjes integrals, Differentiation under integral sign

Uniform and absolute convergence of sequences and series of functions, Uniform convergence and continuity: Term by term differentiation and integration, Improper integrals and their convergence; their absolute and uniform convergence.

### **Recommended Books:**

1. Donald W. K., An Introduction to the Point Set and Algebraic Areas, The Williams & Wilkins Company, 2013.
2. Mattuck, A, Introduction to Analysis, Pearson, 1998.
3. Mendelson B., Introduction to Topology, Dover Publications, 3<sup>rd</sup> Edition, 2010.
4. Munkers, James R., Topology, Pearson Education, 2000.
5. Thomson B.S., Bruckner J. B., Bruckner A.M., Elementary Real Analysis, 2<sup>nd</sup> Edition. 2008.

### **M-503 Abstract Algebra (3 + 0)**

**Group theory:** Groups, Subgroups, Cyclic groups, Normal subgroup, Quotient groups, example, Homomorphisms of groups, fundamental theorems of group homomorphisms, the group isomorphism theorems, Internal and external direct products of groups, Finitely generated Abelian groups, Generators and torsion, The fundamental theorem of Abelian groups, Group action on a set, Fixed sets and isotropy subgroups, Orbits, Sylows theorems, P-group, First, second and third isotropy theorems, Application of the Sylow theory, Nilpotent and solvable subgroup, Normal and subnormal series of subgroups.

**Ring Theory:** Definition and examples of rings, Unit, prime and irreducible elements in a commutative ring with identity, Zero divisors, Ideals, Principal Ideals, Prime ideals, Maximal ideals, Quotient rings, Integral domain, Polynomial rings, The characteristic of a ring, Ring homomorphism, Fundamental theorems of ring homomorphisms, Matrix rings, Rings of endomorphisms, Concept of divisibility in rings, Unique factorization domain (UFD), Principal ideal domain (PID), Euclidean domain.

#### **Recommended Books:**

1. Dummit D.S., and Foote R.M., Abstract Algebra, 3<sup>rd</sup> edition, Addison-Wesley Publishing Company, 2004.
2. Fraleigh J.B., A First Course in Abstract Algebra, Addison- Wesley Publishing Company, 2002.
3. Mattuck, A, Introduction to Analysis, Pearson, 1998.
4. Smith J.D.H., Introduction to Abstract Algebra, 2<sup>nd</sup> edition, Chapman, and Hall CRC Press, 2015.
5. Tom Judson's Abstract Algebra: Theory and Applications (open textbook) 2017.
6. Rauf Querashi M.A., Foundation of Abstract Algebra, 1<sup>st</sup> Edition, 2018.

### **M-505 Numerical Analysis-I ( 2 + 1 )**

Error Analysis, Solution of nonlinear algebraic equations, Newton-Raphson, Graeffe's root squaring and Muller's methods, Bairstow's method for quadratic factors, Quotient Difference Algorithm, Numerical Solutions of linear algebraic equations, Jacobi's, Gauss-Seidel and Relaxation Methods, LU decomposition methods.

**Interpolation:** Newton forward and divided difference formula, Lagrange Interpolation with applications, Splines Interpolation with different end conditions.

**Numerical integration and differentiation:** Trapezoidal rule; Simpson's rule, Romberg, integration, gauss quadrature, improper integrals, Richardson extrapolation, derivatives of unequally spaced data.

#### **Lab/Practical: (Use any software/Language)**

1. Calculation of Error analysis and precession in software
2. Basic programming: List and string manipulation and Mathematical formula
3. Plotting algebraic and transcendental functions/equations
4. Programming for solving algebraic and transcendental equations by Bisection, Newton-Raphson and False position methods
5. Programming for solving system of algebraic equations (Cramer's, Gaussian Elimination and Jordan methods)
6. Programming and Simulation for finding the equi-spaced interpolation Methods
7. Analyze Spline Curves with different end conditions
8. Programming for Numerical Integration methods (Trapezoidal and Simpson rules)

#### **Recommended Books:**

1. Burden.R.L. Faires J.D., Numerical Analysis, Brooks/Cole Publishing Company, 2000, 2<sup>nd</sup> edition.
2. Gerald C.F., Wheatley P.O., Applied Numerical Analysis, Pearson Education, Singapore, 2005, 7<sup>th</sup> edition.
3. Howard J.P., Computational Methods for Numerical Analysis with R, CRC Press, 2017.
4. Jain M.K., Iyenger M.R.K., Jain R.K., Numerical Methods for Scientific and Engineering Computation, New Age International 2003.
5. Sheldon M. Ross, Introduction to probability and statistics for engineers and scientists, 5<sup>th</sup> edition.

### **M-507 Differential Geometry (3 + 0)**

Notation, conventions and recapitulations of vector space theory, Affine space and subspaces; hyper planes, Affine coordinate transformations, Affine maps, Smooth curves and functions on affine spaces, Tangent vectors; directional derivatives and derivations, Tangent space.

**Theory of Space Curves:** Introduction, index notation and summation convention. Space curves, arc length, tangent, normal and binomial Osculating, normal and rectifying planes, Curvature and torsion, The Frenet-Serret theorem, Natural equation of a curve, Involutives and evolutes, helices Fundamental existence theorem of space curves

**Theory of Surfaces:** Coordinate transformation, Tangent plane and surface normal. The first fundamental form and the metric tensor, the second fundamental form, Principal, Gaussian, mean, geodesic and normal curvatures, Gauss and Weingarten equations, Gauss and Codazzi equations, Visualization of curves and surfaces in software, Surfaces, Differential geometry on surfaces, Riemannian geometry, Curvature, Surface geometry in terms of exterior forms, Levi-Civita connection Covariant derivative, Connection and curvature.

#### **Recommended Books:**

1. Abbena E., Salamon S., Gray A., Modern Differential Geometry of Curves and Surfaces with Mathematica, 3<sup>rd</sup>edition, Chapman and Hall/CRC 2006.
2. Banchoff T.F., Lovett S.T., Differential Geometry of Curves and Surfaces, 2<sup>nd</sup>edition, Chapman and Hall/CRC, 2015.
3. Kreyzig E., Differential Geometry, Dover, 1991.
4. Kuhnel W., Differential Geometry: Curves – Surfaces – Manifolds. Student Mathematical library, vol.16. Providence, RI: American Mathematical Society, 2002.
5. Michael S., Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus,

### **M-509 Methods of Mathematical Physics ( 3 + 0 )**

Differential equations and mathematical modeling, Classification of differential equations, Solutions, Linear differential equations a superposition principle, Boundary value and initial value problems, Dynamical system; their analysis and control, Existence, uniqueness a stability of solutions.

Function spaces; orthogonal sets of Functions and generalized Fourier series, Sturm Liouville Theory, Periodic function, periodic extension, even and odd functions, Fourier coefficients, Expansion of function in Fourier series, Function with arbitrary periods, Fourier sine and cosine series.

Linear, ordinary differential equations of order  $n > 1$  (choose  $n=2,3$ ) Existence and uniqueness theorem (statement and application only), Wronskian and fundamental sets of solution, System of differential equations, Methods of solution: reduction of order, undetermined coefficients, variation of parameter and Greens function, Power series solution

Partial differential equations of Mathematical Physics; Method of separation of variables, Boundary value problems relating vibration of strings and membranes, heat conduction and potential theory.

#### **Recommended Books:**

1. Adams, Malcolm R., Victor G., Measure Theory and Probability. Birkhäuser, 1996.
2. Boyce W.E., Elementary Differential Equations, 11<sup>th</sup>Edition, John Wiley and Sons, 2005.
3. Brown J.W., Churchill R.W., Fourier series and Boundary Value Problems, McGraw Hill, 2006.
4. Haberman R., Applied Partial Differential Equations: With Fourier series and Boundary Value Problems. 4<sup>th</sup>Edition. Prentice Hall, 2003.
5. Powers D.L., Boundary Value Problems and Partial Differential Equations, 5<sup>th</sup>Edition, Academic Press, 2005.

## *Sixth Semester*

### **M-502 Complex Analysis (3 + 0)**

**Functions of Complex Variables:** Definition, limit and continuity, Branches of functions, Differentiate and analytic functions The Cauchy-Riemann equations, Entire functions, Harmonic functions, and Elementary functions: The exponential, Trigonometric, Hyperbolic, Logarithmic and Inverse elementary functions, Open mapping theorem, Maximum modulus theorem.

**Complex Integrals:** Contours and contour integrals, Cauchy-Goursat theorem, Cauchy integral formula, Liouville's theorem, Morera's theorem.

**Series:** Power series, Radius of convergence and analyticity, Taylor's and Laurent's series, Integration and differentiation of power series

Singularities, Poles and residues: Zero, singularities. Poles and Residues, Types of singular points, Calculus of residues, contour integration, Cauchy's residue theorem with applications. Mobius transforms, Conformal mappings and transformations.

Expansion of Functions and Analytic Continuation: Mittag-Leffler theorem, Weierstrass's factorization theorem, Analytic continuation.

Elliptic Functions Periodic functions, Elliptic functions and its properties, Weierstrass function, Differential equation satisfied by Integral formula for  $(z)$ , Addition theorem for  $(z)$ , Duplication formula for  $(z)$ , Elliptic functions in terms of Weierstrass function with the same periods, Quasi periodic functions: The zeta and sigma functions of Weierstrass, Jacobian elliptic functions and its properties.

### **Recommended Books:**

1. Ahlfors, Lars V. Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable. 3<sup>rd</sup>edition. New York, NY: McGraw-Hill, 1979.
2. Brown J.W., Churchill, Complex Variables and Applications, 7<sup>th</sup>edition, McGraw Hill Company, 2004.
3. Kasana H.S., Complex Variables: Theory and Applications, Prentice Hall, 2005, 2<sup>nd</sup>edition.
4. Rudin, Walter. Principles of Mathematical Analysis (International Series in Pure and Applied Mathematics), 3<sup>rd</sup>edition. McGraw-Hill, 1976.
5. Zill D.G., Shanahan P.D., Complex Analysis, Jones & Barlett Publishers, 2003.

### **M-504 Topology ( 3 + 0 )**

**Number Systems:** Ordered fields, Rational, real, and complex numbers, Archimedean property, supremum, infimum and completeness,

**Metric Spaces:** Definition and various examples of metric spaces, Holder's inequality, Cauchy-schwarz and Minkowski's inequality, Open and closed balls, Neighborhoods, Open and closed sets, Interior, Exterior and boundary points, Limit points, Closure of a set, Convergence in metric spaces, Cauchy sequences, Continuity in metric spaces, Inner product and norm, Orthonormal sets and basis, The Gram-Schmidt process

**Topology:** Topology of real numbers: Convergence, completeness, completion of real numbers, Open sets, closed sets, compact sets, Heine-Borel Theorem, Connected sets, Sequences and Series of Real Numbers: Limits of sequences, algebra of limits, Bolzano Weierstrass Theorem, Cauchy sequences, liminf, limsup, Limits of series, convergences tests, absolute and conditional convergence Power series

### **Recommended Books:**

1. Bredon, Glen E. Topology and Geometry (Graduate Texts in Mathematics). Springer-Verlag Berlin and Heidelberg GmbH & Company, 1993
2. Donald W.K., An Introduction to the Point Set and Algebraic Areas, The Williams & Wilkins Company, 2013.
3. Mendelson B., Introduction to Topology, Dover Publications, 3<sup>rd</sup> Edition, 2010.
4. Munkres, James. Topology. 2<sup>nd</sup> Edition. Upper Saddle River, NJ: Prentice Hall, 28 December 1999.
5. Rudin, W. Principles of Mathematical Analysis. 3<sup>rd</sup> Edition. McGraw-Hill Science/Engineering/Math, New York, NY: McGraw-Hill, 1976.

## **M-506 Numerical Analysis-II (2 + 1)**

**Difference Equations** (Homogeneous and non-homogeneous): Formation, Solutions directly and by Z-Transform.

**Ordinary differential equations** (ODE): Taylor Series method, Euler and modified Euler methods, Runge-Kutta methods, Predictor-Corrector Method, Milne's and Adams-Bashforth methods, derivations of the formulae, convergence criteria, Errors and error propagation.

**Partial Differential equations** (PDE): (Parabolic, hyperbolic and elliptic.), Laplace, Heat and wave Equations, Crank-Nicolson Method, Alternating Direction Implicit (ADI) Schemes.

### **Lab/Practical: (Use any software/Language)**

1. Symbolic Computation of Difference Equations
2. Plotting and the solutions of difference equations
3. Programing (for solving ODE): Euler's, Improved Euler and Modified Euler's methods
4. Programing (for solving ODE): Taylor's, Runge-Kutta methods
4. Programing (for solving ODE): Predictor-Corrector methods
5. Programing (for solving PDE): Laplace equation with Dirichlet boundary
6. Programing (for solving PDE): Laplace equation with Neumann boundary
7. Programing (for solving PDE): Heat Equation
8. Programing (for solving PDE): Heat Equation (Crank-Nicolson Method)
9. Programing (for solving PDE): Wave Equation
10. Programing for BVP (Shooting method)

**Recommended Books:** Same as in M-505

## **M-508 Functional Analysis-I (3 + 0)**

**Metric Space:** Review of metric spaces, Convergence in metric spaces, complete metric spaces, Dense sets and separable spaces, No-where dense sets, Baire category theorem

**Normed Spaces:** Normed linear spaces, Banach spaces, Equivalent norms, Linear operator, Finite dimensional normed spaces, Continuous and bounded linear operators, Dual spaces

**Inner Product Spaces:** Definition and examples, Orthonormal sets and bases, Annihilators, projections, Linear functionals on Hilbert spaces, Reflexivity of Hilbert spaces

### **Recommended Books:**

1. Conway J.B., A Course in Functional Analysis, 2<sup>nd</sup> edition, Springer-Verlag, Berlin, 1997.
2. Dieudonné, History of Functional Analysis, volume 49, 1<sup>st</sup> edition, 1983.
3. Kreyszig E., Introduction to Functional Analysis with Applications, John Wiley and Sons, 2004.
4. Michael R. B. S., Methods of Modern Mathematical Physics, vol. I: Functional Analysis, 1981.
5. Yosida K., Functional Analysis, 5<sup>th</sup> edition, Springer-Verlag, Berlin, 1995.



## **M-510 Integral Transforms and Special Functions ( 3 + 0 )**

**Fourier transforms:** Introduction, The Fourier Integral Formula, Definition and problems of Fourier Transforms  
Laplace transforms: Introduction, Definition, properties and applications, Gibbs Phenomenon, Inverse Laplace Transforms, Solutions of ordinary and partial differential equations, Initial and boundary value problems.

**Hankle transforms:** Introduction, properties and applications, Solutions of Partial differential equations, Initial and boundary value problems.

**Z-transforms:** Introduction, Definition of Z-Transforms, Inverse Z-Transforms, Solutions of difference equations.  
The Special Functions: Bernoulli Numbers and Polynomials, Euler Numbers and Polynomials, Stirling Numbers, The Gamma function, Important Properties, Infinite Products, Logarithmic Derivative of the Gamma Function, Riemann's Zeta Function, Asymptotic Expansions.

Legendre and Bessel's equations, Properties of Legendre-polynomials and Bessel's Functions, The Legendre Differential Equation, Ordinary Legendre Functions, The Chebyshev polynomials, properties and relations

### **Recommended Books:**

1. Brychkov Y.A., Handbook of Special Functions, Derivatives, Integrals, Series and Other Formulas, Chapman and Hall/CRC Press, 2019.
2. Compos L.M.B.C., Singular Differential Equations and Special Functions, CRC Press, 2019.
3. Debnath L., Bhatta D., Integral Transforms and their Applications, 3<sup>rd</sup> edition, Chapman and Hall/CRC Press 2014.
4. Kreyszig E., Advanced Engineering Mathematics, 8<sup>th</sup> edition, John Wiley & Sons, 2006.
5. Michael R. B. S., Methods of Modern Mathematical Physics, vol. I: Functional Analysis, 1981.

### **Pure Mathematics**

SEMESTER VII			SEMESTER VIII		
Course #	Course Title	Cr.Hr	Course #	Course Title	Cr.Hr
M-601	Theory of Rings & Modules	3+0	M-602	Galois Theory and its Applications	3+0
M-605	Measure Theory-I	3+0	M-606	Measure Theory-II	3+0
M-611	Functional Analysis-II	3+0	M-612	Algebraic Topology	3+0
	Optional-I	3+0		Optional-II	3+0
	Optional-I	3+0		Optional-II	3+0
M600.1	Field Experience / Internship	0+3	M600.2	Capstone Project	0+3
		<b>Total Credit Hours</b>			<b>Total Credit Hours</b>
		<b>18</b>			<b>18</b>

### **Applied Mathematics**

SEMESTER VII			SEMESTER VIII		
Course #	Course Title	Cr. Hr	Course #	Course Title	Cr.Hr
M-651	Classical Mechanics-I	3+0	M-652	Classical Mechanics-II	3+0
M-655	Fluid Dynamics-I	3+0	M-656	Fluid Dynamics-II	3+0
M-653	Tensor Analysis	3+0	M-654	Partial Differential Equations	3+0
	Optional-I			Optional-II	
	Optional-I			Optional-II	
M600.1	Field Experience / Internship	0+3	M600.2	Capstone Project	0+3
		<b>Total Credit Hours</b>			<b>Total Credit Hours</b>
		<b>18</b>			<b>18</b>

## Seventh Semester

### **M-601 Theory of Rings and Modules (3 + 0)**

**Rings:** Review of rings and its types, polynomial rings, set of all polynomial over a ring, Principal ideal ring, embedding of a ring, polynomial rings over unique factorization domain, Euclidean rings, Eienstien's criterion of irreducibility, Fundamental theorems of ring homomorphisms, Ring with chain condition, Noetherian rings and domains, Rings of fraction, Rings with ORE conditions, Artinian rings.

**Modules:** Definition and examples of modules over ring (commutative ring with identity), Submodules, Operations on submodules, generation of modules, finitely generated modules, Direct sum of modules, Cyclic modules, Free modules, Quotient modules, Homomorphisms of modules, Isomorphism theorems of modules, Short exact sequences of modules, Group of module homomorphisms, Simple modules, Modules over a PID, Artinian rings and modules, Noetherian rings and modules, Modules of finite length.

#### **Recommended Books:**

1. Allenby, R.B.J.T, Rings, Fields, and Groups: An Introduction to Abstract Algebra, Edward Arnold Ltd., 1983.
2. Dauns J, Zhou Y., Classes of Modules, CRC Press, 2006.
3. Farb P., Dennis R., Noncommutative Algebra (Graduate texts in Mathematics) Springer, 1993.
4. Fraleigh, J.B.: A First course in Abstract Algebra, 3<sup>rd</sup> Edition, Addison Wesley Publishing Co., 1982
5. Tom Judson's Abstract Algebra: Theory and Applications (open textbook) 2017.

### **M-605 Measure Theory-I ( 3 + 0)**

Measurable Sets: Outer measure, Lebesgue measure, Lebesgue measurable sets, Borel sets, Non measurable sets

Measurable Functions: Lebesgue measurable functions, Simple functions, characteristic functions, Borel measurable function, Littlewood three principles.

The Lebesgue Integration: Review of the Riemann integral, Lebesgue integral, Integral of a non-negative function, Integral of measurable functions, Convergence in measure

#### **Recommended Books:**

1. Adams, Malcolm Ritchie, and Victor Guillemin. Measure Theory and Probability. Birkhäuser, 1996.
2. Goffman C., Pedeick, G., Firsts Course in Functional Analysis, Prentice Hall, 1965.
3. Halmos P.R., Measure Theory, Van Nostrand 1950.
4. Kentelman H., Modern theories of Integration, Dover, 1980.
5. Royden H., Fitzpatrick P., Real Analysis 4<sup>th</sup> Edition.

### **M-611 Functional Analysis-II (3 + 0)**

Compact Normed Spaces: Completion of metric spaces, Completion of normed spaces, Compactification, Nowhere and everywhere dense sets and category, Generated subspaces and closed subspaces, Factor Spaces, Completeness in the factor spaces

Complete Orthonormal set: Complete orthonormal sets, Total orthonormal sets, Parseval's identity, Bessel's inequality

The Specific geometry of Hilbert Spaces: Hilbert spaces, Bases of Hilbert spaces, Cardinality of Hilbert spaces, Linear manifolds and subspaces, Othogonal subspaces of Hilbert spaces, Polynomial bases in  $L_2$  spaces

Semi-norms, Semi norms, locally convex Spaces, Quasi normed linear spaces, Bounded linear functionals, Hahn Banach theorem

Conjugate spaces: Second conjugate space of  $p$   $l$ , The Riesz representation theorem for linear functionals on a Hilbert spaces, Conjugate space of  $C[a,b]$ , A representation theorem for bounded linear functionals on  $C[a,b]$

Uniform Boundedness: Weak convergence, The Principle of uniform boundedness, Consequences of the principle of uniform boundedness, Graph of a mapping and closed graph theorem.

#### **Recommended Books:**

1. Conway J.B., A Course in Functional Analysis, 2nd ed., Springer-Verlag, Berlin, 1997.
2. Dieudonné, History of Functional Analysis, 1<sup>st</sup> Edition, 1983.
3. Kreyszig E., Introduction to Functional Analysis with Applications, John Wiley and Sons,
4. Michael Reed Barry Simon, Methods of Modern Mathematical Physics, Functional Analysis, vol. 1, 1981.
5. Yosida K., Functional Analysis, 5th ed., Springer-Verlag, Berlin, 1995.

### **M-600.1 Field Experience / Internship (0+3)**

Students are required to complete a supervised field experience or internship at an appropriate organization. This practical component provides students with the opportunity to apply their pure/applied mathematics knowledge to real-world challenges, develop professional competencies, and gain valuable industry exposure.

## **Eighth Semester**

### **M-602 Galois Theory and its Applications (3 + 0)**

The theory of fields, field extension, algebraic extensions, monomorphisms of algebraic extension, Test for irreducibility, Eisenstein's criterion, Other methods of establishing irreducibility, Ruler and compass construction, Splitting fields, the extension of monomorphism with examples, The algebraic closure of a field, Normal extensions, Separability, Galois extensions, Differentiation, Frobenius monomorphism, Inseparable polynomials, Automorphism and fixed fields, The Galois group of a polynomial, The fundamental theorem of Galois theory, the theorem on natural irrationalities.

#### **Recommended Books:**

1. Artin, E., Galois Theory, University of Dam Press, Indiana, 1964.
2. David A. Cox, Galois Theory, 2nd Edition, Wiley.
3. Garling, D.J.H., A Course in Galois Theory, C.U.P., 1986.
4. Serre J.P., Topics in Galois Theory, Chapman and Hall, 2<sup>nd</sup> Edition, 2007.
5. Stewart I.N., Galois Theory, Chapman and Hall, 4<sup>th</sup> Edition, 2015.

### **M-606 Measure Theory-II ( 3 + 0)**

The  $L^p$  Spaces, Constructing Measures, Lebesgue Measure, Cantor Set Experiments Lebesgue Stieljes Measure, Modes Of Convergence, Decomposition of Measures, Connections To Riemann Integration, Fubini Type Results, Differentiation

#### **Recommended Books: same as M-603**

### **M-612 Algebraic Topology (3 + 0)**

Homotopy and Homotopy types, Cell Complexes, Operations on Spaces, Two Criteria for Homotopy Equivalence, The Homotopy Extension Property, Paths and Homotopy, The Fundamental Group of the Circle, Induced Homomorphisms, Free Products of Groups, The van Kampen Theorem, Applications to Cell Complexes

CW-complexes, delta-complexes, simplicial homology, exact sequences, diagram chasing, Singular homology, homotopies and chain homotopies, categories and functors, Eilenberg-Steenrod axioms, Excision, computations for spheres, equivalence of simplicial and singular homology, Cellular homology, Mayer-Vietoris sequences, the Mayer-Vietoris argument, homology with coefficients, Tensor products, Tor, universal coefficient theorem for homology, products of simplices, The Eilenberg-Zilber shuffle "product" map, diagonal approximations, the Alexander-Whitney map, method of acyclic models, Kunneth formula, Duality, cohomology, Ext, universal coefficients for cohomology, Projective spaces and Grassmannians, cup products, relative cup products, Dual Kunneth formula, field coefficients, cup products in cohomology of projective spaces,

#### **Recommended Books:**

1. Goffman C., Pedeick, G., Firsts Course in Functional Analysis, Prentice Hall, 1965.
2. Hatcher, Allen. Algebraic Topology, Cambridge, UK: Cambridge University Press, 2002.
3. May. A Concise Course in Algebraic Topology. Chicago, IL: University of Chicago Press, 1999.
4. Rotman, Joseph J., An Introduction to Algebraic Topology, New York, NY: Springer-Verlag, 1998.
5. Shastri A.R., Basic Algebraic Topology, Chapman and Hall/CRC Press, 2013.

### **M-600.2 Capstone Project (0+3)**

Students will undertake an in-depth capstone project to apply their pure mathematics expertise to a complex problem or innovative solution. This culminating experience involves rigorous research, development, and documentation, culminating in a comprehensive report and presentation. The project is designed to foster critical thinking, problem-solving, and the ability to translate theoretical knowledge into practical applications.

## **Applied Mathematics (Compulsory Courses)**

### **M-651 Classical Mechanics-I (3 + 0)**

**Lagrangian Dynamics:** Coordinate system, degree of freedom, Constraints (Holonomic and non-holonomic), Generalized coordinates, D'Alembert Principle, Lagrangian equations from D'Alembert Principle.

**Hamiltonian dynamics:** Generalized momentum and cyclic coordinates, Conservation theorems, Linear and angular momentum, Jacobi integral, Hamilton's equations, Example of Hamiltonian Dynamics: Oscillator, Motion of a particle in central force, Charge particle, Compound pendulum.

**Two body central force:** central force and motion in a plane, Equation of orbit, Kepler's law.

**Calculus of variations:** The Euler-Lagrange Equations, The principle of least action

**Canonical Transformations:** Canonical and Legendre Transformations, applications, bilinear invariant transformation.

### **Recommended Books:**

1. Goldstein H., Classical Mechanics, Addison Wesley, 1962.
2. Reinhard H., Classical Mechanics (Including an Introduction to the Theory of Elasticity), Springer 2020.
3. Rund H., The Hamilton Jacobi Theory in the Calculus of Variations, D. Van Nostrand, 1966.
4. Taylor E.F., Wheeler J.A., Space-time Physics, W. H. Freeman, 1965.
5. Victor I., Lectures in Classical Mechanics (With Solved Problems and Exercises) Springer 2020.

### **M-655 Fluid Dynamics-I (3+0)**

Introduction, Fluid and its properties, Streamline, Pathline, and Streakline, Conservation Principles for a Material Region, Basic Analysis Techniques, Some Interesting Flows, Properties of Velocity Field, Vorticity and Its Substantial Derivative.

Control Volume Analysis: Lagrangian versus Eulerian Approach, Reynolds Transport Theorem, Integral Mass Conservation Equation, Differential Mass Conservation, Equation, Linear Momentum Equation in Inertial Reference Frame, Linear Momentum Equation in Non-Inertial Reference Frame, Angular Momentum Equation in Inertial and Non-Inertial Reference Frames, Energy Conservation Equation, Entropy Equation

Potential Flows: Basic Concepts, Elementary Plane Potential Flows, Superposition of Two or More Plane Potential Flows, Force and Moment on a Body in Plane Potential Flows, Conformal Transformation, Circular cylinder without circulation, Circular cylinder with circulation, Blasius theorem, Kutta condition and the flat-plate airfoil, Joukowski airfoil, Vortex motion, Karman's vortex street, Method of images, Velocity potential, Stoke's stream function, Solution of the Potential equation, Uniform flow, Source and sink, Flow due to a doublet

Navier–Stokes Equations Forces on a Fluid Element, Deformation Rate Tensor, Differential Forms of the Equations of Motion, Navier–Stokes Equations, Exact Solutions, Navier–Stokes Equations in Terms of Vorticity and Stream Function, Flow between rigid bodies.

### **Recommended Books:**

1. Acheson D.J., Elementary Fluid Dynamics, Clarendon Press, Oxford, 1990.
2. Bansal J.I., Viscous Fluid Dynamics, Oxford & IBH Publishers Co, New Delhi, 1977.
3. Emanuel G., Analytical Fluid Dynamics, CRC Press, 2015.
4. Munson B.R., Young D.F., Okiishi, T.H., Fundamentals of Fluid Mechanics, John Wiley Sons, N.Y., 1994.
5. O'Neill M.E., Cholton F., Ideal and Incompressible Fluid Dynamics, Ellis Borwood Ltd, West Sussex, England, 1986.

### **M-653 Tensor Analysis (3 + 0)**

Coordinate transformations, Einstein summation convention, Tensors of different ranks, Contravariant, covariant and mixed tensors, Symmetric and skew symmetric tensors, Addition, subtraction, inner and outer products of tensors, Contraction theorem, quotient law, The line element and metric tensor, Christoffel symbols.

Riemannian connections and geodesics: Introduction, Affine connections, Riemannian connections, Geodesics, The exponential map, Minimizing properties of geodesics

Curvature: Introduction, The curvature tensor, The second fundamental form, Sectional and Ricci curvatures, Jacobi fields, Manifolds of constant curvature, Applications of Tensors in General Theory of Relativity, Tensors in Continuum Mechanics.

### **Recommended Books:**

1. Altman W., De Oliveira A.M., Physical Components of Tensors, 1<sup>st</sup> Edition, CRC Press, 2014.
2. Chorlton F., Vector and Tensor Methods, Ellis Horwood Publisher, U.K., 1977.
3. Joshi A.W., Matrices and Tensors in Physics, Wiley Eastern Limited, 1991.
4. Somasundaram D., Differential Geometry, Narosa Publishing House, New Delhi. 2005.
5. Spiegel M.R., Vector Analysis, McGraw Hill Book Company, Singapore, 1981.

### **M-652 Classical Mechanics-II (3+0)**

Brackets and Liouville's bracket, Poisson and Lagrange brackets, Relations between brackets, angular momentum and Poisson bracket, Contact transformations, Hamilton-Jacobi equations, Small Oscillation: Potential energy and equilibrium, Two coupled oscillator, General theory and Small Oscillation, Dynamics of a rigid body: Generalized coordinate and rigid body, Euler's angle, Moment of Inertia, Symmetric bodies, Motion of a heavy symmetrical top, Relativistic Mechanics: Mass-Energy relation, Conservation laws, Lorentz transformation, Minkowski space.

### **Recommended Books: same as M-651**

### **M-656 Fluid Dynamics-II (3+0)**

Navier–Stokes Equations Exact solution of Navier–Stokes Equations, Poiseuille flow, Couette flow, Flow in a cylinder, Flow between two cylinders, Stoke's first and second problems and others.

Dimensional Analysis: Methods for Dimensional analysis, Theorems and its applications

Compressible Flow Classification of Compressible Flows, Compressible Flow, Functions, Variable-Area Duct Flow with Friction, Heat Transfer, and Rotation, Isentropic Flow in a Variable-Area Duct, Isentropic Flow in a Constant-Area Duct with Rotation, Isentropic Flow in a Variable-Area Duct with Rotation, Fanno Flow, Rayleigh Flow, Isothermal Constant-Area Flow with Friction, Normal Shock, Oblique Shock, Prandtl–Meyer Flow, Operation of Nozzles and Diffusers

Open channel flows: Methods and formulae for different channels

### **Recommended Books: same as M-655**

### **M-654 Partial Differential Equations (3 + 0)**

Introduction, formation of PDE, Solution, Equation  $dx/P=dy/Q=dz/R$ , Linear PDE first order, Lagrange's form, Solutions by direct integration, Linear homogenous PDE with constant coefficient, Complementary function, Particular integrals, General PDE of nth order, Charpit's method, Jacobi's method, non-Homogenous equations, separation of variable, PDE of order two with variable coefficients, Laplace transformation, Canonical forms, Linear hyperbolic equation, Monge's methods, PDE in Engineering and physics, Solutions by Fourier series, Vibrations of strings, wave equation, heat equation, one and two dimensions, Transmission line equations, telegraph equations, radio equations, signal equations, vibrations of a rod, Boundary value problems.

### **Recommended Books:**

1. Haberman R., Elementary Applied Partial Differential Equations, 2nd edition, Prentice Hall Inc., New Jersey, 1987.
2. Krantz S.G., Differential Equations Theory, Technique and Practice, Second Edition, CRC Press, 2015.
3. Olver, Peter. Introduction to Partial Differential Equations. Springer, 2013
4. Sneddon N., Elements of Partial Differential Equations, Dover Publishing, Inc., 2006.
5. Yakubov Y., Yakubov S., Differential-Operator Equations: Ordinary and Partial Differential Equations, CRC Press, 2014.

## **Optional subject PURE MATHEMATICS and APPLIED MATHEMATICS**

### **M-663 Introduction to Computer Programming (2+1)**

Introduction to GNU-Linux and bash commands., Introduction to (K) UBUNTU as a personal OS for the PC, The KDE Windowing interface, and utilities, specially, Kate, k3b, Firefox, chrome, terminal, wicd, Setting up users, and maintaining a list of users and super users. Bash commands: ls, rm, cd, mkdir, cp, scp, rsync, ssh, chmod, chown, less, more, head, tail, ping, nslookup, ps, source, Setting up batch files as executable text file containing valid Linux commands. Setting up path (system wide vs. for particular users), installing python, installing gcc, Introduction to Programming with MATLAB/ Octave as a training platform, MATLAB keywords, commands, script M-files, popular toolboxes, working with scalars and matrices, saving the MATLAB session, using the input command, using the fprintf command, directing fprintf output to a file.

#### **Lab / Practical (Programming with MATLAB)**

Assigning variables, rules for variable names, using to suppress echo (output on screen), branching using if, else, else-if structures, and their nested forms, Using for loops and while loops to control repetition, loops in nested form. The break - continue commands. Infinite loops and how to circumvent them, Function M-files, and user-defined functions in MATLAB, anonymous functions, anonymous functions as input to other functions, Small programming project the uses all of these concepts.

#### **Recommended Books:**

1. French C.S., Computer Science, Cengage Learning, 2007.
2. Nortons P., Introduction to Computers, 6<sup>th</sup> Edition, McGraw-Hill, 2004.
3. W Hajek Darrell Cesar Herrera, Introduction to Computers, Create Space Independent Publishing Platform, 2017
4. Wempen Faithe, Computing Fundamentals: Introduction to Computers, Sybex, 2015
5. Williams B.K., Sawyer S.C., Using Information Technology, A Practical Introduction to Computers & Communications, Board Book 2015.

### **M-664 Programming in Python (2+1)**

Intro and Overview, Principle of Software Engineering and Reusing and Extending Code, Review of Fundamentals of Procedural Programming, Objects, Data Abstraction, Information Hiding & Encapsulation, Constructors, destructors, and object creation, Name space and references, Class Methods, Methods Overloading, Inheritance, Polymorphism, Abstract Classes, Exceptions, Exception Handling, Templates, Example translations of concepts in Python to Java, Practical Example: Data Science Classes, Student final project presentations.

**Labs:** Relevant problem on each topic of Python Programming using Python

#### **Recommended books**

1. Ayeva, K., & Kasampalis, S. (2018). Mastering Python Design Patterns: A guide to creating smart, efficient, and reusable software, 2<sup>nd</sup> Edition, Packt Publishing.
2. Charles R. Severance. (2016). Python for Everybody: Exploring Data in Python 3, University of Michigan
3. Goldwasser, M. H., & Letscher, D. (2008). Object-oriented Programming in Python, Pearson Prentice Hall.
4. Lott, S. F. (2019). Mastering Object-Oriented Python: Build powerful applications with reusable code using OOP design patterns and Python 3.7, 2<sup>nd</sup> Edition: Packt Publishing.
5. Lott, S. F., & Phillips, D. (2021). Python Object-Oriented Programming: Build robust and maintainable object-oriented Python applications and libraries, 4<sup>th</sup> Edition: Packt Publishing.

### **M-615 Summability Theory-I ( 3 + 0 )**

Metric transformations, ToeplitzSteinhaus and Kojima Schur theorems and their Integral analogues, Regularity, consistency, equivalence and inclusions of Bob methods of summability

### **M-616 Summability Theory-II ( 3 + 0 )**

Holder and Cesaro's means, Mercer'e theorem, Summability of integrals, Euler Borel and Hausdorff methods of summability

#### **Recommended Books: (for M-615 &M-616)**

1. Basar F., Dutta H., Summable Spaces and Their Duals, Matrix Transformations and Geometric Properties, CRC Press, 2020.
2. Hardy, G.H., Divergent Series, Clarendon Press, 1977.
3. Knopp K., Theory and Applications of Infinite Series, Blakie& sons 1990.
4. Natarajan P.N., Sequence Spaces and Summability over Valued Fields, CRC Press, 2019.
5. Peterson G.M., Regular Matrix Transformations, McGraw Hill, 1966.

### **M-647 Projective Geometry-I ( 3 + 0 )**

Projective properties of conics, Chasle's theorem, projective generation of the conic, Homographic correspondences on the conics, Pascal's theorem, Linear systems of conics, Relation to Euclidean geometry, Parallel Postulate, Pythagorean Theorem and Thales Theorem.

### **M-648 Projective Geometry-II ( 3 + 0 )**

Projective geometry of three dimensions, point, straight line and plane, Duality, class ratios, Quadric surfaces, Line geometry, Cross-Ratio and Direct Linear Transformation, Fubini-Study Metric and Grassmannian, Twisted cubic, Homogeneous Coordinates and Incidence.

#### **Recommended Books: (for M-647 and M-648)**

1. Fairley L., Mabry D., Euclidean and Projective Geometry, World Technologies, 2014.
2. Maxwell E.A., Methods of Plane Projective Geometry, C.U.P., 1952.
3. Maxwell E.A., General Homogenous Co-ordinates in Space of Three Dimensions, C.U.U. 1960.
4. Sample J.G., Kenton G.T., Algebraic Projective Geometry, C.U.P. 1955.
5. Wylie C.R., Introduction to Projective Geometry, Dover Publications, 2008.

### **M-661 Electromagnetics-I ( 3 + 0 )**

Coulomb's law, electrostatic field and potential, Gauss law, Conductor and condensers, capacity, energy and force in electrostatic field, Dielectrics, Boundary value problems of electrostatics, Method of images and methods of Laplace and Poisson equations

### **M-662 Electromagnetics-II ( 3 + 0 )**

Steady currents, Magnetic fields of currents, vector potential, Magnetic materials and permanent magnetism, Electromagnetic induction, Maxwell's equations, Reflection, Refraction and Diffraction, Transmission Lines, Wave guides and Cavity Resonators, Radiation Mechanism, Eddy Currents

#### **Recommended Books: (for M-661 and M-662)**

1. Chambers L.G., An Introduction to the Mathematics of Electricity and Magnetism, Chapman Hall, 1973.
2. Coulson, C.A., Electricity, 5<sup>th</sup> edition, Oliver and Boyd, 1965.
3. David H. Staelin, Ann W. Morgenthaler, Jin Au Kong, Electromagnetic Waves, Pearson, 1<sup>st</sup> edition, 1993.
4. Ferraro V.C.A., Electromagnetic Theory, Athlone, 1957.
5. Lorrain P., Corson D.R., Introduction to Electromagnetic Fields and Waves 2<sup>nd</sup> edition, W.H. Freeman, 1970.

### **M-671 Relativity-I ( 3 + 0 )**

Principles of special relativity, Lorentz transformations and consequences, review of modern differential geometry curvature tensor, Ricci tensor, Bianchi identity, Weyl tensor, geodesics, geodesic deviation, acceleration in terms of curvature tensor, Einstein field equation, Quantum Einstein field equations, Schwarzschild solutions, other solutions, Predictions of general relativity.

### **M-672 Relativity-II ( 3 + 0 )**

Solutions of Einstein field equations other than Schwarzschild solutions. Non-relativity black holes, maximal extension and conformal compactification, charged black holes, rotating black holes, Linearized theory of gravity, Cosmography, Newtonian cosmology, Hubble's law, cosmological principle, relativistic cosmology.

#### **Recommended Books : ( For M-671 and M-672)**

1. Pathria R., The Theory of Relativity, 2<sup>nd</sup> edition, Pergamon, London.
2. Resnick, Robert. Introduction to Special Relativity. New York, NY: Wiley, 1968.
3. Synge, J.L., Relativity: The Special Theory, North Holland, Amsterdam. 1980.
4. Synge. J.L., Relativity: The General Theory, North Holland, Amsterdam, 1975.
5. Taylor, Edwin F., and John A. Wheeler. Exploring Black Holes: Introduction to General Relativity. San Francisco, CA: Addison Wesley Longman, 2000.

### **M-685 Astronomy-I ( 3 + 0 )**

Introduction, Naked Eye Observations, Ancient World Models, Observations Made by Instruments, The Nature of the Observables, The Astronomer's Measurements, The Night Sky, The celestial sphere and elementary celestial mechanics, The Geometry of the Sphere, The Celestial Sphere: Coordinate Systems, The Celestial Sphere: Timekeeping Systems, The Reduction of Positional Observations: The Reduction of Positional Observations: Geocentric Planetary Phenomena, Celestial Mechanics: The Two-Body Problem, Celestial Mechanics: The Many-Body Problem, Spherical trigonometry, fundamental formulas, solution of spherical triangles, duality.

### **M- 686 Astronomy-II ( 3 + 0 )**

The Radiation Laws, The Optics of Telescope Collectors, Visual Use of Telescopes, Detectors for Optical Telescopes, Astronomical Optical Measurements, Modern Telescopes and Other Optical Systems, Radio Telescopes, Telescope Mountings, High Energy Instruments and Other Detectors, Planetary phenomena, direct and retrograde motion, synodic periods, brightness and phases of planet, Aberration, precession, nutation, parallax and their effects on positions of stars, Eclipse and occultation, Binary and variable stars, H.R. diagram, nebula, the galaxy summer constellations. Sextant, theodolite, Use of star maps and almanac.

#### **Recommended Books: (for M-685 and M-686)**

1. Abell G.O., Exploration of the universe, Holt Rinehart and Winston.
2. Baker R.H., An Introduction to Astronomy, D. Van Nostrand, 1940.
3. Smart W.M., Text Book on Spherical Astronomy, C.U.P.
4. Weinberg, Steven. The First Three Minutes: A Modern View of the Origin of the Universe. 2<sup>nd</sup> updated edition, Basic Books, 1993.
5. Zeilik, Michael, and Stephen A. Gregory. Introductory Astronomy and Astrophysics. 4th edition Fort Worth, TX: Saunders College Publishing, 1997.



### **M-631 Numerical Methods ( 3 + 0 )**

Numerical double integration, Approximation theory: Rational Approximation, Pade Approximation, Chebyshev and Legendre approximation, properties, Gaussian Quadrature, Lobatto, Radu, Chebyshev and Hermite Integration methods, Collocation, Galerkin's method, Rayleigh-Ritz method, Finite Element Method, Approximate evaluation of eigen values and eigen functions, Aitken process, Sturm sequences, Bairstow's method, Q-B Schemes, Lohmer-Schur method, Numerical solutions of nonlinear system Newton's method, Steepest-descent method, Homotopy continuation method.

#### **Recommended Books:**

1. Allen III. M.B., Isaacs E.L., Numerical Analysis for Applied Sciences (Pure and Applied Mathematics: Wiley-Interscience series texts), John Wiley & Sons Inc, N.Y. 1998.
2. Atkinson K.E., An Introduction to Numerical Analysis, John Wiley & Sons, 1989.
3. Burden, Richard L., and J. Douglas Faires. Numerical Analysis. 7<sup>th</sup> edition, Belmont, CA: Brooks Cole, 2000
4. Jain M.K., Iyengar S.R.K., Jain R. K., Computational Methods for Partial Differential Equations. Wiley Eastern Limited, New Delhi, 1991.
5. Jain M.K., Iyengar, S.R.K., Jain R.K., Numerical Methods for Scientific and Engineering computations. Wiley Eastern Limited, New Delhi, 1991.

### **M-632 Integral Equations ( 3 + 0 )**

Linear integral equations of the first kind, Linear integral equations of the second kind, Relation between differential and integral equations, Relationship between differential equation and Volterra integral equation Neumann series, Fredholm Integral equation of the second kind with separable kernels, Eigenvalues and eigenvectors, Iterated functions, Quadrature methods, Least square methods, Homogeneous integral equations of the second kind, Fredholm integral equations of the first kind, Fredholm integral equations of the second kind, Abel's integral equations, Hilbert Schmidt theory of integral equations with symmetric kernels Regularization and filtering techniques.

#### **Recommended Books:**

1. Baker C.T.C, Integral Equations, Clarendon Press. 1977.
2. Kanwal, R. P. Linear Integral Equations: Theory and Technique. Boston, MA: Birkhauser, 1996.
3. Masujima, M. Applied Mathematical Methods in Theoretical Physics - Integral Equations and Calculus of Variations. Weinheim, Germany, Wiley-VCH, 2<sup>nd</sup> edition, 2009.
4. Stakgold, I. Green's Functions and Boundary-value Problems. New York, NY: Wiley, 1998.
5. Wazwaz A.M., A first Course in Integral Equations, World Scientific, 1989.

### **M-633 Mathematical Statistics-I ( 3 + 0 )**

Probability theory, Sampling theory, Order Statistics, Point estimation, Interval estimation, Limiting distributions, Method of Moments, Estimators, Maximum Likelihood Estimators, Bayes Estimators

### **M-634 Mathematical Statistics-II ( 3 + 0 )**

Sufficient Statistics, Statistical hypothesis and tests, Analysis of variance, Multivariate normal distribution, Distribution of quadratic form, Advanced methods and statistical software.

#### **Recommended Books: (for M-633 and M-634)**

1. Ang, Alfredo, and Wilson Tang. Probability Concepts in Engineering Planning and Design: vol.1, Basic Principles. New York, NY: John Wiley & Sons, 1975.
2. Fisz. M., Probability Theory and Mathematical Statistics, John Wiley & sons, 1963.
3. Hogg R.V., Craig A.T., Introduction to Mathematical Statistics, 4<sup>th</sup> edition, Collier Macmillan, 1978.
4. Kendall M.G., Stuart A., The Advanced Theory of Statistics, Charles Griffin, vol.3, 4<sup>th</sup> edition, 1985.
5. Lindgren B.W., Statistical Theory, Macmillan.

## **M-637 Applied Algebra-I (2+1)**

Lattices: Introduction to lattice theory, weight and length reduced lattice bases, Application of lattices, Boolean Algebra, Boolean Polynomials, Boolean Rings, Finite Fields and Polynomial congruence's field theory, field extension, Nonlinear polynomial congruence's, Degree of a finite field, complexity of operation in a finite field, Nonlinear polynomial congruence's, Degree two congruence's, Quadratic residues, Legendre symbol and its properties, the law of quadratic reciprocity.

Coding Theory: Linear, cyclic, Goopa Turbo and space time codes, Algebraic coding theory with MAPLE, BCH code with MAPLE, perfect codes and uniformly packed codes, Lloyd theorem, Codes over  $Z_4$ .

### **Lab/Practical: (MAPLE / MATLAB)**

- Introduction to basic lattice structures and how to define them using MAPLE / MATLAB
- Example: Construct a simple 2D lattice and visualize it.
- Implement basic lattice operations such as meet and join
- Visualize the orthogonalized basis vectors.
- Construct a finite field of prime order and examine its lattice structure.
- Explore the connection between Boolean algebra and lattice structures using MAPLE / MATLAB
- Explore the Shortest Vector Problem (SVP) and its implications in cryptography.
- Exploration of linear coding methods
- Study of cyclic codes and their algebraic structure.
- Understanding advanced coding techniques.
- Implementing algebraic codes using MAPLE software.
- Study and application of BCH codes with MAPLE.
- Investigation of perfect and uniformly packed codes.
- Examination of codes defined over the ring  $Z_4$ .

## **M-638 Applied Algebra-II (2+1)**

Cryptography: Classical cryptography, Algebra cryptography with MAPLE, shift substitution, affine Vigenere, Hill, permutation and stream cipher, Variants of RSA system and beyond, Exchanging private keys, El-Gamal cryptosystem with MAPLE, cryptography and elliptic curves with MAPLE, cryptographic in a group, Algebraic curves in a numerical affine plane, lines and rational curves, Hyper-elliptic curves, Elliptic curves, Group law on elliptic curves, Hyper-elliptic curves, Elliptic curves, Group law on elliptic curves. Elliptic curves over  $\mathbb{R}$ ,  $\mathbb{C}$  and  $\mathbb{Q}$ , Elliptic curves over finite fields, Elliptic curves and cryptography.

Polya Theory: Group actions, Burnside theorem, Pattern inventory, switching function with MAPLE Shannon theory, Perfect secrecy, Entropy, cryptography and real world, Application, Young experiment, Quantum computers.

Groups: Fast adding, Polya's theory of enumeration, symmetry group, semi groups and automata semi groups and its applications in real world.

### **Lab/Practical: (MAPLE / MATLAB)**

- Implement the Caesar cipher (a type of shift substitution) in MAPLE / MATLAB
- Encrypt and decrypt messages using different shift values.
- Explore the Affine cipher by implementing encryption and decryption
- Implement the Vigenère cipher and analyze its effectiveness against frequency analysis
- Implement the Hill cipher using matrix operations
- Encrypt and decrypt messages, exploring the impact of different key matrices
- Explore permutation ciphers by implementing a simple transposition
- Implement a stream cipher (e.g., the XOR cipher) and discuss its use in real-world applications
- Explore RSA key generation, encryption, and decryption processes
- Implement variants of RSA, such as RSA with padding schemes
- Analyze the security and efficiency of these variants
- Implement the Diffie-Hellman key exchange protocol
- Simulate a secure key exchange between two parties
- Implement the ElGamal encryption and decryption algorithm
- Explore the mathematical foundations and security of the ElGamal system
- Define elliptic curves over various fields ( $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\mathbb{Q}$ , finite fields)
- Explore the group law on elliptic curves and visualize point addition.
- Calculate entropy and mutual information for various cryptographic systems
- Analyze real-world cryptographic protocols and systems using the principles of Shannon theory.
- Discuss the impact of quantum computing on current cryptographic methods.

### **Recommended Books: (for M-637 and M-638)**

1. Andr'e N.B., Jugen F. B., Volker K., Coding theory, Algorithms, Architectures and application, John Willy and Sons Ltd 2007.
2. Brikhoff, Bartee G.C., Modern Applied Algebra, New York, McGraw Hill 1970.
3. Brikhoff G., Lattice theory. Providence, RI Amer. Math Soc. 1967.
4. Rudolf L., Gunter Pilz, Applied Abstract Algebra, Springer 2<sup>nd</sup> edition 1997.
5. Vanlint J.H., Introduction to coding theory, 3<sup>rd</sup> edition 1998.

### **M-645 Operations Research-I ( 3 + 0 )**

Introduction to operations research: The Origins and Applications of Operations Research, System Modeling Principles.

Linear programming: The Linear Programming Model, The Art of Problem Formulation, Graphical Solution of Linear Programming Problems, Preparation for the Simplex Method, The Simplex Method, Initial Solutions for General Constraints, Two-phase simplex method and Big-M technique, Duality and Sensitivity Analysis, Primal-Dual Relationship, The Dual Simplex Method, Parametric Linear Programming, The Upper Bound Technique, Revised Simplex method and Computational Efficiency, Guide to Software Tools.

The Transportation Problem: A Streamlined Simplex Method for the Transportation Problem, The Assignment Problem, Hungarian method.

Network optimization models, The Shortest-Path Problem, The Minimum Spanning Tree Problem, The Maximum Flow Problem, The Minimum Cost Flow Problem, The Network Simplex Method.

### **M-646 Operations Research-II ( 3 + 0 )**

Project Management with PERT/CPM: Using a Network to Visually Display a Project, Scheduling a Project with PERT/CPM, Dealing with Uncertain Activity Durations, Considering Time-Cost Trade-Offs, Scheduling and Controlling Project Costs.

Dynamic Programming: Characteristics of Dynamic Programming Problems, Some applications.

Integer Programming: Some BIP Applications, Innovative Uses of Binary Variables in Model Formulation, The Branch-and-Bound Technique and Its Application to Integer Programming, Cutting-plane method.

Game Theory: The Formulation of Two-Person, Zero-Sum Games, Games with Mixed Strategies, Graphical Solution Procedure, Solving by Linear Programming.

Decision Analysis: Decision Making without Experimentation, Decision Making with Experimentation, Decision Trees, The Practical Application of Decision Analysis

Queuing Theory: Basic Structure of Queuing Models, The Role of the Exponential Distribution, The Birth-and-Death Process, Queuing Models Based on the Birth-and-Death Process.

Inventory Theory: Components of Inventory Models, Deterministic Continuous-Review Models, A Deterministic Periodic-Review Model

### **Recommended Books: (for M-645 and M-646)**

1. Hillier F.S., Lieberman G., Introduction To Operations Research, 11<sup>th</sup> edition, McGraw-Hill Education, 2019.
2. Taha H.A., Operations Research: An Introduction, 8<sup>th</sup> edition, Pearson Publications. 2017.
3. Taha H.A., Natarajan A.M., Balasubramine P., Tamilarasi A., Operations Research 8<sup>th</sup> edition, Pearson education 2009.
4. Winston, W. L., Operations Research: Applications and Algorithms 4<sup>th</sup> edition, Cengage Learning, 2003.
5. Winston W.L., Venkataramanan M., Introduction to Mathematical Programming: Operations Research, Vol. 1 4<sup>th</sup> Edition, Cengage Learning, Inc 2003.

### **M-649 Stochastic Processes ( 3 + 0 )**

Stochastic Modeling, Probability Review, The Major Discrete and Continuous Distributions, Conditional Probability and Conditional Expectation, Conditioning on a Continuous Random Variable, Martingales, Stochastic Processes,

**Markov Chains:** Chapman-Kolmogorov Equations, Classification of States of a Markov Chain, Long-Run Properties of Markov Chains, First Passage Times, Absorbing States, Markov Decision Processes, Optimal Policies, Policy Improvement Algorithm for Finding Optimal Policies, Discounted Cost Criterion

**Poisson Processes,** The Poisson Distribution and the Poisson Process, The Law of Rare Events, Distributions Associated with the Poisson Process, The Uniform Distribution and Poisson Processes, Spatial Poisson Processes, Compound and Marked Poisson Processes.

**Continuous Time Markov Chains:** Pure Birth Processes, Pure Death Processes, Birth and Death Processes, The Limiting Behavior of Birth and Death Processes, Birth and Death Processes with Absorbing States, Finite State Continuous Time Markov Chains

### **M-650 Renewal Processes & Theory of Queues (3 + 0 )**

**Renewal Processes:** Renewal Phenomena, Definition of a Renewal Process and Related Concepts, Some Examples of Renewal Processes, The Poisson Process Viewed as a Renewal Process, The Asymptotic Behavior of Renewal Processes, Generalizations and Variations on Renewal Processes

**Queuing Theory,** Basic Structure of Queuing Models, Examples of Real Queuing Systems, The Role of the Exponential Distribution, The Birth-and-Death Process, Queuing Models Based on the Birth-and-Death Process, Queuing Models Involving No exponential Distributions, Priority-Discipline Queuing Models, Queuing Networks characteristics of queuing systems,

**The Application of Queuing Theory,** Examples, Decision Making, Formulation of Waiting-Cost Functions, Decision Models, Some Real-life Applications of Queuing Theory

**Simulation:** The Essence of Simulation, Some Common Types of Applications of Simulation, Generation of Random Numbers, Generation of Random Observations from a Probability Distribution, Solving Queuing Problems by Simulation, Performing Simulations on Spreadsheets.

### **Recommended Books: (for M-649 and M-650)**

1. Cinder, E., Introduction to Stochastic Processes, Prentice Hall Inc.1975.
2. Ross S.M., Applied Probability Models with Optimisation Applications, Holder Day Inc. 1970.
3. Taylor H.M., Karlin S., An Introduction to Stochastic Modeling 3rd Edition Academic Press,
4. Karlin S., Pinsky M., An Introduction to Stochastic Modeling 4th Edition Academic Press
5. 1998.  
2010.

### **M-657 Nonlinear systems-I ( 3 + 0 )**

Introduction to nonlinear systems, Bifurcation and difference equations, Classification of bifurcation of equilibrium points in one and in higher dimensions, Difference equations, Stability of fixed points and of periodic solutions, Attractors and volumes, The Logistic equation, Dimension and Fractals, Feigenbaum theory of scaling, Liapunov exponents.

### **M-658 Nonlinear Systems-II ( 3 + 0 )**

Ordinary differential equations, Hamiltonian systems, Stability of periodic solutions, Second order autonomous differential systems, Direct method of Liapunov, Limit cycles, Van der Pol equation, Forced oscillation, Duffing equation, Routes of chaos.

### **Recommended Books: (for M-657 and M-658)**

1. Bazaraa, Mokhtar S., Hanif D. Sherali, and C. M. Shetty. Nonlinear Programming: Theory and Algorithms. New York: John Wiley & Sons, 1993.
2. Jackson E.D.A., Perspective of Nonlinear Dynamics, CUP, 1991.
3. Strogatz S.H., Nonlinear Dynamics and Chaos, 2<sup>nd</sup> edition, CRC Press, 2015.
4. Thompson, J.M.T., Stewart H.B., Nonlinear Dynamics and Chaos. John Wiley 1986.

### **M-687 Advanced Statistics-I ( 3 + 0 )**

Definition of Bayesian Statistics: Bayesian Statistics, Probability distribution, True Distribution, Statistical model, prior, and posterior, Examples of Posterior Distributions, Estimation and Generalization, Marginal Likelihood or Partition Function, Conditional Independent Cases.

Statistical Models: Normal Distribution, Multinomial Distribution, Linear regression, Neural Network, Finite Normal Mixture, Nonparametric Mixture.

Basic Formula of Bayesian Observables: Formal Relation between True and Model Normalized Observables, Cumulant Generating Functions, Basic Bayesian Theory, Regular Posterior Distribution: Division of Partition Function, Asymptotic Free Energy, Asymptotic Losses, Proof of Asymptotic Expansions, Point Estimators.

Standard Posterior Distribution: Standard Form, State Density Function, Asymptotic Free Energy, Renormalized Posterior Distribution, Conditionally Independent Case.

General Posterior Distribution Bayesian Decomposition, Resolution of Singularities, General Asymptotic Theory, Maximum A Posteriori Method Markov Chain Monte Carlo: Metropolis Method, Basic Metropolis Method, Hamiltonian Monte Carlo, Parallel Tempering, Gibbs Sampler, Gibbs Sampler for Normal Mixture, Nonparametric Bayesian Sampler, Numerical Approximation of Bayesian Observables, Generalization and Cross Validation Losses, Numerical Free Energy.

### **M- 688 Advanced Statistics-II ( 3 + 0 )**

Information Criteria Model Selection, Criteria for Generalization Loss, Comparison of ISCV with WAIC, Criteria for Free Energy, Discussion for Model Selection, Hyper parameter Optimization, Criteria for Generalization Loss, Criterion for Free energy, Discussion for Hyper parameter Optimization.

Topics in Bayesian Statistics Formal Optimality, Bayesian Hypothesis Test, Bayesian Model Comparison, Phase Transition, Discovery Process, Hierarchical Bayes, Basic Probability Theory Delta Function, Kullback-Leibler Distance, Probability Space, Empirical Process, Convergence of Expected Values, Mixture by Dirichlet Process

### **Recommended Books: (for M-687 and M-688)**

1. Donovan T., Mickey R.M., Bayesian Statistics: A step-by-step approach, Oxford Scholarship online, 2019.
2. Drake, A. Fundamentals of Applied Probability Theory. New York, NY, McGraw-Hill, 1988.
3. Jeremy J Foster, Emma Barkus., Understanding and Using Advanced Statistics: A Practical Guide for Students, SAGE Publications Ltd; 1<sup>st</sup> edition, 2006.
4. Wantanabe S., Mathematical Theory of Bayesian Statistics, 1<sup>st</sup> edition, CRC Press, 2018.
5. Westfall Peter, Kevin S. S. Henning., Understanding Advanced Statistical Methods; 1st edition, Chapman and Hall/CRC,2013.

### **M-691 Biological Modeling ( 3 + 0 )**

Introduction: The Nature and Purposes of Biological Modeling, The Modeling Process, Types of Mathematical Models, Assumptions, Simplifications, and Compromises, Scale and Choosing Units  
Difference Equations (Discrete Dynamical Systems) Graphical Analysis, Qualitative Analysis and Population Genetics, Intraspecific Competition, Harvesting, Period Doubling and Chaos, Structured Populations, Predator-Prey-Systems.

First-Order Differential Equations (Continuous Dynamical Systems) Continuous-Time Models and Exponential Growth, Logistic Population Models, Graphical Analysis, Equations and Models with Variables Separable, Mixing Processes and Linear Models, First-Order Models with Time Dependence  
Nonlinear Differential Equations Qualitative Analysis Tools, Harvesting, Mass-Action Models, Parameter Changes, Thresholds, and Bifurcations, Numerical Analysis of Differential Equations  
Systems of Differential Equations Graphical Analysis: The Phase Plane, Linearization of a System at an Equilibrium, Linear Systems with Constant Coefficients, Qualitative Analysis of Systems  
Topics in Modeling Systems of Populations Epidemiology: Compartmental Models, Population Biology: Interacting Species, Numerical Approximation to Solutions of Systems  
Systems with Sustained Oscillations and Singularities Oscillations in Neural Activity, Singular Perturbations and Enzyme Kinetics, HIV - An Example from Immunology, Slow Selection in Population Genetics, Second-Order Differential Equations: Acceleration.

#### **Recommended Books:**

1. Adleman, L. M., Molecular computation of solutions to combinatorial problems, Science 266, no. 5187 1994.
2. Brauer F., Kribs C., Dynamical Systems for Biological Modelling: An Introduction, CRC Press 2016.
3. Haefner J.W., Modeling Biological Systems: Principles and Applications, Springer, 2005.
4. Kremling A., Systems Biology: Mathematical Modeling and Model Analysis, CRC Press 2014.
5. Schwartz R., Biological Modeling and Simulation, MIT Press, 2008.

### **M-692 Discrete Maps ( 3 + 0 )**

The Stability of One-Dimensional Maps, Introduction, Maps vs. Difference Equations, Maps vs. Differential Equations, Linear Maps/Difference Equations, Fixed (Equilibrium) Points, Graphical Iteration and Stability, Criteria for Stability, Periodic Points and Their Stability, The Period-Doubling Route to Chaos, Applications. Attraction and Bifurcation Basin of Attraction of Fixed Points, Basin of Attraction of Periodic Orbits, Singer's Theorem, Bifurcation, Sharkovsky's Theorem, The Lorenz Map, Period-Doubling in the Real World, Poincaré Section/Map.

Chaos in One Dimension Density of the Set of Periodic Points, Transitivity, Sensitive dependence, Definition of Chaos, Cantor Sets, Julia set, Symbolic Dynamics, Conjugacy, Other Notions of Chaos, Rössler's Attractor, Saturn's Rings.

Stability of Two-Dimensional Maps Linear Maps vs. Linear Systems, Computing An, Fundamental Set of Solutions, Second-Order Difference Equations, Phase Space Diagrams

Stability Notions, Stability of Linear Systems, The Trace-Determinant Plane, Liapunov Functions for Nonlinear Maps, Linear Systems Revisited.

#### **Recommended Books:**

1. Gilmore R., Lefranc M., The Topology of Chaos: Alice in Stretch and Squeezeland, Wiley 2003.
2. Holmgren R., A First Course in Discrete Dynamical Systems, Springer, 1996.
3. Kulenovic M.R.S., Merino O., Discrete Dynamical Systems and Difference Equations, CRC Press, 1<sup>st</sup> edition, 2002.
4. R. Clark Robinson ., An Introduction to Dynamical Systems: Continuous and Discrete, 2<sup>nd</sup> edition, American Mathematical Society, 2012.
5. Wikan Arild, Discrete Dynamical Systems: with an Introduction to Discrete Optimization Problems, bookboon, 2000.

### **M- 689 Mathematical Finance-I ( 3 + 0 )**

Introduction to financial derivatives, Financial derivatives—what’s the big deal?

Stylized facts, Interest rates, Cash flows, continuously compounded interest rates, Interest rate options: caps and floors

Discrete-Time Finance The binomial one period model, The one period model, The multi period model, Linear Time Series Models Linear systems in the time domain, Linear stochastic processes, Linear processes with a rational transfer function, Auto covariance functions,

Prediction in linear processes, Non-Linear Time Series Models The aim of model building, Qualitative properties of the models, Parameter estimation, Parametric models, Model identification, Prediction in non-linear models, Applications of non-linear models.

Kernel Estimators in Time Series Analysis Non-parametric estimation, Kernel estimators for time series, Kernel estimation for regression, Applications of kernel estimators

Stochastic Calculus Dynamical systems, The Wiener process, Stochastic Integrals, Itô stochastic calculus, Extensions to jump processes, Stochastic differential equations, Analytical solution methods, Feynman–Kac representation, Girsanov measure transformation

### **M- 690 Mathematical Finance-II ( 3 + 0 )**

Continuous-Time Security Markets From discrete to continuous time Classical arbitrage theory, Modern approach using martingale measures, Pricing, Model extensions, Computational methods

Stochastic Interest Rate Models, Gaussian one-factor models, A general class of one-factor models, Time-dependent models, Multifactor and stochastic volatility models

The Term Structure of Interest Rates Basic concepts, The classical approach, The term structure for specific models, Heath–Jarrow–Morton framework, Credit models, Estimation of the term structure—curve-fitting

Discrete-Time Approximations Stochastic Taylor expansion, Convergence, Discretization schemes, Multilevel Monte Carlo, Simulation of SDEs

Parameter Estimation in Discretely Observed SDEs High frequency methods, Approximate methods for linear and non-linear models, State dependent diffusion term, MLE for non-linear diffusions, Generalized method of moments (GMM), Model validation for discretely observed SDEs, Inference in Partially Observed Processes Exact filtering, Conditional moment estimators, Kalman filter, Approximate filters, State filtering and prediction, The unscented Kalman filter, A maximum likelihood method, Sequential Monte Carlo filters, Application of non-linear filters

### **Recommended Books: (for M-689 and M-690)**

1. Robert J., Modelling Fixed Income Securities and Interest Rate Options, 3<sup>rd</sup> edition, Chapman Hall/CRC Press, 2019.
2. Baxter, Martin, and Andrew Rennie. Financial Calculus: An Introduction to Derivative Pricing. Cambridge University Press, 1996
3. Wilmott, Paul, Sam Howison, and Jeff Dewynne. The Mathematics of Financial Derivatives: A Student Introduction. Cambridge University Press, 1995
4. Fanelli V., Financial Modelling in Commodity Markets, 1<sup>st</sup> edition, Chapman Hall/CRC Press, 2019.
5. Lindström E., Madsen H., Nielsen J.N., Statistics for Finance, CRC Press, 2015.



### **M- 693 Optimization Theory ( 3 + 0 )**

Statement of the problem, condition for optimality, concept of direction of search, alternating direction and steepest descent methods, conjugate direction method, conjugate gradient method, Newton's method, Quasi-Newton equation, derivation of updating formulae for Quasi-Newton's equation, The Gauss-Newton method, The Levenberg-Marquardt method, The corrected Gauss-Newton method, Methods for large scale problems. Theory of constrained optimization, methods for minimizing a general function subject to linear equality constraints, active set strategies for linear inequality constraints, special forms of the objectives functions, Lagrange multiplier estimates, Changes in working set, Barriers function methods, Penalty functions methods, Methods based on Lagrangian functions reduced gradient and gradient projection methods.

#### **Recommended Books:**

1. Bertsimas, Dimitris, and John Tsitsiklis. Introduction to Linear Optimization. Belmont, MA: Athena Scientific, 1997.
2. Fletcher R., Practical Methods of Optimization, vol.I& II, John Wiley and Sons, 1980.
3. Gill P.E., Murray E., Wright, H.H., Practical Optimization, Academic Press, 1981.
4. Gotfreid B.S., Weisan J., Introduction to Optimization Theory, Prentice Hall, Englewood Cliffs, NJ, USA, 1973.
5. Luenberger D.G., Optimization by Vector Space Methods, John Wiley & Sons, 1986.

### **M- 694 Machine Learning and Data Science (2 + 1)**

Importing, Summarizing, and Visualizing Data Introduction, Structuring Features According to Type, Visualizing Data, Plotting Qualitative Variables, Plotting Quantitative Variables, Data Visualization in a Bivariate Setting Statistical Learning, Supervised and Unsupervised Learning, Training and Test Loss, Tradeoffs in Statistical Learning, Estimating Risk, In-Sample Risk, Cross-Validation, Modeling Data, Multivariate Normal Models, Normal Linear Models, Bayesian Learning

Monte Carlo Methods, Monte Carlo Sampling, Generating Random Numbers, Simulating Random Variables, Simulating Random Vectors and Processes, Resampling, Markov Chain Monte Carlo, Monte Carlo Estimation, Crude Monte Carlo

Bootstrap Method, Variance Reduction, Monte Carlo for Optimization, Simulated Annealing, Cross-Entropy Method, Splitting for Optimization.

Regression: Linear Regression, Analysis via Linear Models, Parameter Estimation, Model Selection and Prediction, Cross-Validation and Predictive Residual Sum of Squares, In-Sample Risk and Akaike Information Criterion, Nested Models, Coefficient of Determination, Inference for Normal Linear Models, Comparing Two Normal Linear Models, Confidence and Prediction Intervals, Linear Models in Python

**Lab/Practical:** Visualizing Data, Qualitative Variables, Data Science Getting Started, Python Objects, Types and Operators, Functions and Methods, Modules, Flow Control, Iteration, Classes, Files, NumPy, Creating and Shaping Arrays, Slicing, Array Operations, Random Numbers, Matplotlib, Creating a Basic Plot, Pandas, Series and Data Frame, Manipulating Data Frames, Extracting Information, Plotting, Scikit-learn, Partitioning the Data, Standardization, Fitting and Prediction, Testing the Model.

#### **Recommended Books:**

1. EMC Education Services, Data science and big data analytics: discovering, analyzing, visualizing and presenting data, Wiley, 2015
2. Gutierrez D.D., Machine Learning and Data Science: An Introduction to Statistical Learning Methods with R, Technics Publications, 1<sup>st</sup> edition, 2015.
3. Kroese D.P., Botev Z., Taimre T., Vaisman R., Data Science and Machine Learning Mathematical and Statistical Methods, Chapman and Hall/CRC, 2019.
4. Ratner B., Statistical and Machine-Learning Data Mining: Techniques for Better Predictive Modeling and Analysis of Big Data, Chapman and Hall/CRC 3<sup>rd</sup> edition, 2020.
5. Zhang, Y., New advances in machine learning. Pearson, 2010

## **M- 695 Computational Number Theory ( 3 + 0 )**

Number Theory: Preliminaries, Well-ordering principle, Principle of finite induction

Divisibility theory: The division algorithms, Basis representation theorem, Prime and composite numbers, Canonical decomposition, The greatest common divisor, The Euclidean algorithm, The fundamental theorem of arithmetic, Least common multiple

Linear Diophantine equations: Congruence's, Linear congruence's, System of linear congruence's, The Chinese remainder theorem, Divisibility tests, Solving polynomial congruence's, Fermat's and Euler's theorems, Wilson's theorem

Arithmetic of Finite Fields Existence and Uniqueness of Finite Fields, Representation of Finite Fields, Implementation of Finite Field Arithmetic, Some Properties of Finite Fields, Alternative Representations of Finite Fields, Computing Isomorphisms among Representations.

Arithmetic of Polynomials, Polynomials over Finite Fields, Finding Roots of polynomials over Finite Fields, Factoring Polynomials over Finite Fields, Properties of Polynomials with Integer Coefficients, Factoring Polynomials with Integer Coefficients.

Arithmetic of Elliptic Curves What Is an Elliptic Curve? Elliptic-Curve Group, Elliptic Curves over Finite Fields, Some Theory of Algebraic Curves, Pairing on Elliptic Curves, Elliptic-Curve Point Counting.

### **Recommended Books:**

1. Boulagouaz M., Tignol J-P, Algebra and Number Theory, Chapman and Hall/CRC, 1<sup>st</sup> edition, 1999.
2. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers Oxford University Press, 2008.
3. Hardy, G.H., and Edward M. Wright. An Introduction to the Theory of Numbers. Oxford University Press, 1960.
4. Kraft JS, Washington LC, Elementary Number Theory, Chapman and Hall/CRC, 1<sup>st</sup> edition, 2014.
5. Stein William, Elementary Number Theory: Primes, congruence and Secrets, Springer, 2008.

## **M-696 Scientific Computing (2 + 1)**

### **C-Programming Primer with MEX-C interface in MATLAB**

Structure and semantics of C language, definition of code blocks using { }, The main function, and important library headers and their declaration, the include statement, the define statement, Writing a simple Hello World program, The main function, and passing parameters to it, C language syntax for if, switch case, for, while, loops, The goto statement and how / why to avoid it, Mex-C program signature, Memory handling in C, and in Mex-C files, Testing and debugging Mex-C codes, Checking stand-alone C-code for memory leaks using valgrind, Errors in Scientific Computing.

### **Discretization using Finite Difference Methods**

Continuous and discrete boundary value problems, Stencil Notation, Types of PDEs, Grids and Discretization approaches, Poisson's equation, Matrix Terminology, Eigenvalues of continuous Laplace Operator, Exercises.

**Direct Methods** The Gaussian Elimination Method, Norms, and Floating point numbers, Error Analysis of Gaussian Elimination, Pivoting, and iterative improvement, Cholesky decomposition for SPD matrices, Band Matrices, General Sparse Matrices, Exercises.

### **Iterative Methods - Basic**

Iterative Solvers, Splitting's Preconditions, Jacobi and Gauss-Seidel iterative solution methods, Starting vector and termination criteria, convergence of Jacobi and Gauss-Seidel. Exercises.

### **Iterative Methods - Krylov-subspace Methods**

Method for Systems with an SPD matrix, the Chebyshev method, the Conjugate Gradient (CG) method, the convergence behaviour of the CG method. Preconditioning of Krylov Subspace Methods, the Preconditioned Conjugate Gradients (PCG) method. Methods for General Matrices. Indefinite Symmetric matrices, iterative methods for General matrices, CG applied to the normal equations, BiCG type methods. GMRES type methods, Choice of iterative method, preconditioning for General matrices. Exercises.

### **Lab/Practical:**

Usage of MATLAB environment, Functions, Operators, Pseudo-code, flowcharts, and documentation, Designing an algorithm to find roots, Eigenvalues and solving algebraic equations, Mathematical models for problem solving, GUI Interface, basic function of MATLAB and its usage to solving ODE, PDE, Direct and Iterative Methods.

### **Recommended Books:**

1. Attaway, S. MATLAB: A Practical Introduction to Programming and Problem Solving, Butterworth-Heinemann, 3<sup>rd</sup> edition, 2013.
2. Gustafsson, Bertil. ,Fundamentals of Scientific Computing, Springer-Verlag Berlin Heidelberg, 2011.
3. Palm, W., Introduction to MATLAB for Engineers, New York, NY, McGraw-Hill, 3<sup>rd</sup> edition, 2010.
4. Quarteroni, Alfio, Saleri, Fausto., Scientific Computing with MATLAB, Springer-Verlag Berlin Heidelberg, 2004
5. Xue D., Chen Y.Q., Scientific Computing with MATLAB, CRC Press, 2<sup>nd</sup> edition, 2016.

## **M-697 Actuarial Mathematics-I**

The Concept of Interest, Simple, Compound Interest, The Rate of Interest, Nominal Rates of Interest, Accumulation Factors, The Force of Interest, Present Values, Present Values of Cash Flows, Discrete Cash Flows, Continuously Payable, Cash Flows, Valuing Cash, Flows, Interest Income, Capital Gains and Losses, Taxation, Interest Rate Quantities, The Equation of Value, Annuities certain: Present Values and Accumulations, Deferred Annuities, Continuously Payable Annuities, Varying Annuities, Uncertain Payments.

The Economics of Insurance, Utility Theory, Elements of Insurance, Optimal Insurance, Individual Risk Models for a short term, Models for Individual Claim random variables, Approximations for the distribution of the sum, Survival distributions and life tables, Probability for the age at death, The survival function, Time until Death for a person age  $x$ , Curtate Future lifetimes, Force of Mortality, Life table, Survival function, The deterministic survivorship group, Assumptions for fractional ages, Life insurance, Insurances payable at the moment of death, Level Benefit, Endowment, Deffered, Varying benefit insurances, Differential equations for at the moment of death, Life annuities, Continuous and discrete Life annuities,  $m$ thly payment, Benefit premiums, Fully continuous and discrete premiums, Apportionable premiums, Benefit reserves, Fully continuous/discrete benefit reserves.

### **Recommended Books:**

1. Bowers, N.L., Gerber H.U., Hickman J.C., Jones D.A., Nesbitt C.J., Actuarial Mathematics, The society of actuaries, 1997.
2. Broverman S.A., Mathematics of Investment and credit, Actex academy, 5<sup>th</sup> edition, 2010.
3. Garret S.J., An Introduction to the Mathematics of Finance: A Deterministic Approach, Elsevier, 2<sup>nd</sup> edition, 2013.
4. Gupta A.K., Varga T., An Introduction to Actuarial Mathematics, Springer, 2002.
5. V. I. Rotar, Actuarial Models: The Mathematics of Insurance, Second Edition, Chapman and Hall/CRC, 2014

## **M-698 Actuarial Mathematics-II**

Analysis of benefit reserves, Benefit reserves general insurances, Recursion relations for fully Benefit reserves, fractional duration, Differential equations for fully continuous benefit reserves

Multiple life function, Joint distributions of future lifetimes, The joint-life status, The last-survivor status, dependent lifetime models, Common shock, copulas, Insurance and annuity benefits, survival statuses, special two life annuities, reversionary annuities, Simple contingent functions.

Multiple decrement model, Random survivorship group, Uniform distribution assumption for multiple decrement, Application of Multiple decrement theory, Actuarial present value and numerical evaluation, Benefit premium and reserves, valuation of pension plans, Disability benefits with life insurance, demographic assumptions, Waiver of premium benefits.

Collective Risk models for a single period, Distribution of claims, compound distribution, Collective risk models over an extended period, discrete/continuous model, Applications of Risk theory, stop-loss reinsurance, Theory and problems of pension funding.

### **Recommended Books: same as in M-697**

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