Concept Building in the Undergraduate Mathematics and Physics Curricula

Syed Arif Kamal[1,*], Khursheed A. Siddiqui[2] and Naseeruddin[3]

[1]Department of Mathematics, University of Karachi, Karachi-75270, Pakistan; [2]Department of Physics, King Abdul Aziz University, Jeddah, Kingdom of Saudi Arabia and [3]Department of Physics, University of Karachi, Karachi-75270, Pakistan

Abstract: The basic concepts of scientific method should be reinforced in the undergraduate mathematics and physics curricula. Students should be able to distinguish what is science and what is not science. The key is to keep the terminologies to a minimum and to introduce the correct concepts depending on the cognitive abilities of the students. Experiments need to be designed to demonstrate the concepts taught in the classroom and to inculcate scientific thinking.

Key Words: Pedagogical techniques, Concept building.

INTRODUCTION

Science and its study occupy a most important part of today's education. But what is really science and what is not science. Can science answer all of the questions faced by the humankind? Certainly, science has its limitations. It can only deal with what can be observed or felt. It cannot make value judgments. There are also limitations of time to find the answer as well as lack of absolute certainty in the answers. Even well-accepted theories have a dead end somewhere. Consider big bang theory of creation of the universe. What was there before the big bang? How was there such a huge energy concentrated in a small volume? These questions and others shall continue to haunt the mind of the inquiring and the seeker of truth. To quote Freeman Dyson [4] of the Institute of Advanced Study, Princeton, USA: "no matter how far we go into the future, there will always be new things happening, new information coming in, new worlds to explore, a constantly expanding domain of life, consciousness and memory".

Time and again persons from outside the mainstream of science claim that they have made a breakthrough. They have proved one of the established theories to be false. Are they always wrong? Sometimes it happens that persons who do not hold basic degrees in mathematics or physics make major contributions in these subjects. Eugene P. Wigner, Nobel Laureate in Physics and contributor to Nuclear, Particle and Solid State Physics, held an engineering degree. Can anyone overlook his contributions in Physics? What should, then, be the criterion to judge a theory to be seriously considered? Let us see if there are some established rules to seriously consider a theory [6, 7].

*Address correspondence to this author at the Department of Mathematics, University of Karachi, Karachi-75270, Pakistan;
E-mail: kamal@ngds-ku.org

THE METHOD OF DOING SCIENCE

Science could be simulated as a ping-pong game between a theorist and an experimentalist. A theorist tries to understand nature and builds an idea (hypothesis) on the basis of available knowledge and then suggests to an observer to look for certain predictions. Examples of a few hypotheses are:

a) Heavier bodies fall towards earth with larger acceleration.

b) Moon does not have its own light, but glows because of reflection from sunlight.

The following are not scientific hypotheses:

a) Anthony is a good boy.

b) War is bad for humankind.

c) The reward for praying in the first row is more as compared to praying in the second row.

An observer then goes out and sees if the theorist's idea is true. If an idea matches the reality, it becomes a model of the real world. If the idea does not match with the real world, it becomes a conjecture. If an idea becomes a model, then many people check the same idea under different circumstances. If the idea matches the real world in all situations, then it becomes a theory. For example, it took 12 years (1967-1979) for the model of electro-weak interaction to become Glashow-Weinberg-Salam Theory [23]. If a theory predicts, correctly, and if it can be found correct on a larger time scale, then it becomes a natural law. Examples may be cited from classical mechanics. Every one is familiar with

0250-5363/09

the Newton's Laws of Motion and the Law of Universal Gravitation. It implies that a natural law depends on the available information of the physical world and the basic axioms of the mathematics used. A theorist goes on modeling in this way and always works for a better understanding of the natural world. There is always a possibility of change in basic physical information or in the basic axioms of mathematics. Whenever this happens, a theorist searches for new ideas to revolutionize an understanding of the real world.

Can one prove anything in science? No! One can only disprove a statement. A scientific statement is that statement for which an experiment can be devised to disprove it [14]. A theory cannot be called absolutely correct or proving the observed results. All one can say is that a theory is usable under the given set of conditions. If the conditions change or new phenomena are observed the theory needs to be modified. Even the most established laws, *e. g.*, Newton's Law of Universal Gravitation had to be modified when new logics and observations were at our disposal.

A new model presented to the scientific community will be seriously considered if it satisfies one or more of the following conditions — developed and extended from Zeilik [24]:

a) It is *internally consistent* (*i. e.*, mathematically sound).

b) It *explains* known observations.

c) It *predicts* future observations and *suggests* controlled experiments, which could be performed in a laboratory.

d) It *is verifiable* by a number of observations.

e) It is *changeable* to match observations better.

For example, Dirac's theory of the electron was mathematically rigorous, it explained the observed electron spin and it predicted the positive electron later discovered and named as the positron. String theories are internally consistent and they explain a number of phenomena, but they do not propose a test, which can be performed in the laboratory or observed in our environment. Therefore, some people have doubts about them.

These concepts may be taught by not introducing strange terminologies in the beginning. Simple words like "idea"; "idea accepted", "idea rejected", "idea universally accepted" and "idea accepted through ages" may be used in place of hypothesis, model, conjecture, theory and law, respectively. Once the concepts are firmly in place in the minds of the students proper terminologies could be introduced at the end of a lecture.

Students must be trained to think scientifically. The logic of science is not built in the human mind, but it must be inculcated. To see this point ask ten children of class five or six the following question: Drop two balls. One directly to the ground and the other given a small horizontal velocity. Which of the two balls reaches the ground first? You will be surprised to learn that most kids answer that the first ball reaches the ground first. Their perception is that the first ball travels a shorter distance. Then ask them to perform the actual experiment. They will, immediately, see that they were wrong. In fact, both the balls reach the ground at the same time because downward velocity is independent of horizontal velocity.

BUILDING CONCEPTS

A concept is built in four stages: acquisition, formation, development and application. All these stages must be incorporated in the depth and the breadth of curricula. The curriculum, itself, should not be static [10, 11]. It must be, constantly, evolving on the basis of inputs from the students, the peers, the parents and the experts (who should critically review the teaching sessions from videotapes, *etc.*) and must conform to needs of the society it is designed for. No matter how good the curriculum is, it would not serve its purpose unless the teachers are motivated and their own concepts are clear and firm (Summers, 1995). Before teaching a new concept students' perception of the topic must be assessed [22]. Any wrong or distorted concept, already present, must be identified. With the right concepts mapped properly, the students shall be able to relate and apply concepts and techniques learnt in various disciplines and sub-disciplines [17, 18]. For example [13]:

a) Electric and gravitational potential: similarity in mathematical expression

b) Fast freeze in structural biology and fertilizer production: similarity of process

One concept from mathematics and one concept from physics is presented. Possible ways of introducing the concept are suggested:

The Concepts of Deductive and Inductive Logic

If one generalizes from specific observations one is said to be applying inductive logic. For example, the following line of argument is based on inductive logic. Water and oil are liquids. They flow and so all the liquids flow. In basic sciences, one obtains general laws from specific observations applying inductive logic. If one obtains a particular result from a general law one is said to be applying deductive logic. A line of argument that goes as the following is based on deductive logic. All liquids flow. Water is a liquid. Therefore, it must flow. In applied sciences, one develops technology based on the general laws using deductive logic. Here, again the words inductive logic and deductive logic may be avoided by using "generalization from specific observations" and "deducing specific results from a general law".

The Concept of Reproducibility

The observations on which a scientific theory is based must be reproducible by anyone under suitable circumstances [12]. If one takes observations at different places and times one must obtain the same results. Such observations are called stable. Two observers conducting the same experiment under identical conditions must get identical results. Such observations are termed as objective. Not all the observations are identical at different places. If an observer measures atmospheric pressure at Karachi and the other measures it at Peshawar, these values shall be different. Similarly, all the observations are not identical at different times. One observes stars on sky during the night and sun during the day. Are the laws based on these observations valid at different places and different times? Physics assumes that the laws are globally valid. In fact, it does not make any sense to do science, spend a lot of money and time to find laws, which are not valid at different places and different periods of time. If one is not careful one may try to formulate a general law, which does not satisfy these conditions. For example, it never snows in Karachi or Lahore. If one makes a law that it never snows in Pakistan it would be incorrect because it does snow in Muree (law not valid at different places). Else, from the general observation that it is never dark during the day, a law made that it is never dark during the day is wrong because it is dark during a total solar eclipse - law not valid at different times.

The word reproducibility may be avoided in the beginning. One could state that one is interested in finding out if the observations taken by a scientist may be repeated by the same person at some other time and at some other place as well as other scientists trained in a similar way.

THE WRONG OR THE MISSING CONCEPTS

Examples from everyday life help clarify difficult concepts. Care must be exercised that attempts towards simplification should not introduce wrong concepts, which would be difficult to wipe out at a later stage. An example shall illustrate the situation. One of the authors (SAK) introduced the Dirac equation in BSc (Hons), Second Year (equivalent to sophomore in the American system). A simplified but a rigorous treatment was given of the electron spin. The plane- wave solutions were obtained and the negative energies interpreted to bring in the concept of positron. However, the word spinor was, skillfully, avoided throughout the discussion as a proper definition was beyond the scope of that class. A colleague, while discussing solutions of the Dirac equation in the same class, remarked that plane-wave solutions were spinors. He, further, added that the spinor was a quantity, which had 4 components. This was a disaster. A space-time-vector-field formulation (in 4 dimensions) has 4 com-ponents. A tensor may have 4 components. But these quantities are, definitely, not spinors. If the terminology "spinor" was, nevertheless, brought up by a student a proper answer must be that a spinor is a quantity, which has 4 components, and it transforms in a special way under the Lorentz transfor-mations, which is to be taught in higher classes. One, frequently, encounters these high-fi terminologies by over-ambitious students. Great care and trick must be exercised in answering these questions. An enthusiastic student must not be discouraged from asking questions. At the same time, other students should not be left confused and lose interest and, most important of all, no wrong concepts should be imparted during the process.

Most of the time these students have read a book without digesting the material and without building the proper concepts. They may have verified a mathematical equation without understanding the physics behind that equation [19]. The following examples illustrate the situation:

Example 1: A student was doing his MSc thesis in Mathematical Physics, which involved calculation of curvature of the universe. He was asked by one of the authors (SAK) what he understood from curvature. He started mentioning a complicated set of equations. He was, then, asked if he could calculate curvatures of the outer surface of a cylindrical clip holder. The answer was in the negative.

Example 2: Another student, who just got his BE (Bachelor of Engineering)) in Mechanical Engineering was asked by SAK, which of the following situations would result in greater stability against toppling for a passenger-carrying bus:

a) The cargo is loaded on the roof of bus.

b) The cargo is loaded in the space provided under the passenger compartment.

He could not answer the question. It was discovered, later, that the student had no concept of center of gravity.

Example 3: A university physics professor (KAS) went as an external examiner in a local college in Karachi. The student was performing photocell experiment. A couple of meter scales were present on the table. When asked if he had seen a meter scale the answer was in the negative. KAS then asked, "How long is a meter?" The student pointed to the end of the laboratory stating that was the length of a meter. This was the worst of all the situations, illustrating that the student had no concept, even, of the order of magnitudes. The last example is taken from Bukhari & Kamal [3].

Physicists educated in this manner would *not* be able to discover a new law [5, 16]. Engineers educated under this system would *only* perform maintenance and installation. They would *not* be able to make a new system or even adapt an existing system to their needs. One should not be surprised that undergraduate physics education in Pakistan does not include a course in Error Analysis and undergraduate engineering education does not include a course in Experiment Designs. As a result, a physicist cannot design and conduct an experiment properly; an engineer cannot test and maintain quality of the products produced.

The basic problem lies in the pre-university science education [20]. Science curricula must be modified starting from the primary level and there should be continuity through the university level.

THE MATERIAL AND THE METHODS

Basic treatment of the scientific method, the logical reasoning, the modeling and the simulation, the significance of symmetry in simplifying physics problems, the constituents of matter and the fundamental forces of nature were introduced to the entering students in the basic sciences as well as the medical and the engineering colleges. The pre-medical students did not have a calculus background, while the pre-engineering and the basic-science students had a basic knowledge of differentiation and integration. Response of the students was obtained through individual discussions, written evaluations after every lecture and end-of-the-term evaluation. It was found that no prior knowledge of the medical students of certain concepts made it easier to inculcate the right thinking. Hence, the wrong concepts introduced at an early stage may be very difficult to correct later.

One of the authors (SAK) introduced term papers, research proposals, presentations, discussion and problem-solving sessions, daily-lecture feedback from students (through summary of lectures, comments and ways to improve the lectures), evaluation of personality and of background preparation (by asking the students to write a life-history essay, to take a diagnostic test at the beginning of a course), concept of continuous revision, preparation for graduate studies/professional career as well as multiple-format quizzes & tests in the undergraduate courses taught by him. Students' response was overwhelming.

The class, which was hesitant to solve inclined-plane problem (suitable for Class 9 students) at the beginning of the course, was attempting problems from the Physics PhD (Doctor of Philosophy) Preliminary Examinations of the Johns Hopkins University and University of California, Berkeley, within a span of 4 months. A few students co-authored research papers with SAK. Some students got motivated and joined PhD programs in Germany and Singapore. On another occasion, SAK introduced the concept of growth velocity, measurement of children's height, interpretation of the NCHS (National Center for Health Statistics) Growth Charts (the most recent version downloaded from the Internet) to pre-medical students, who just passed their HSC (Higher Secondary Certificate — after passing this examination the student enters a professional-degree program or a baccalaureate-degree program in a university) Examinations. Recall that these students had a class 10 background in mathematics, as pre-medical students in Pakistan do not study mathematics in intermediate. The students learnt to employ the technique of linear interpolation (as application of the slope-intercept form of equation of straight line), and, subsequently, produced research reports [1, 2]. This became possible because the students were highly motivated [8].

THE ROLE OF DISCUSSIONS AND PROJECTS

Discussion and problem-solving sessions have a very important role in clarifying the concepts. In discussion sessions a problem may be treated using a different approach, subtle points considered and students invited to comment on the limitations of a given approach. In fact, students have expressed more interest in discussions as compared to problem-solving sessions. Projects motivate the students [8]. They, also, give students a chance to work in groups, find a problem, propose a tentative solution and carry out investigations on their own all of them contributing to original thinking. A written report accompanied with a public presentation should improve the writing and the communication abilities of a student. A project should not be assigned to an individual. A group of 2-3 students provide an opportunity to work in a team. However, every student may be assigned to tackle a different aspect of a common problem. An open discussion policy with the supervisor and colleagues should be established. Credits must be given for even small suggestions and little contributions from individuals in and outside the working group. Plagiarism must be made a serious offense. As far as possible the students should be encouraged to find a problem and suggest a tentative line of action to handle the problem.

It is better to formulate the problem as an open-ended one. This would produce independence and confidence in the students. Although, the projects have so many merits they are no substitutes for formal course work and formal evaluation. It is found that a project assignment, actually, distracts a student from formal course work, which results in a serious lack of training. To alleviate this problem a project title should be assigned after the final examinations of a term have been completed. Students may work on the project during the term break and submit the report and make a presentation during the first week of next term.

Not more than one project or paper may be assigned during the term break. An academic year may be divided into Fall (September-December), Spring (January-April) and Summer (May-July) Terms. Formal teaching may be conducted during the Fall and the Spring Terms. The Summer Term may be reserved for independent studies and longer projects.

Curricula (starting from the pre-primary level) should be developed considering the breadth (the vocabulary and the concepts acquired in concurrent courses) and the depth (developing the concepts on the basis of previous knowledge in the same and the other subjects). One should give due consideration to the philosophy of teaching a certain topic, the contents of that topic and the pedagogical techniques to be used. With the current state-of-affairs of the educational standards in our state-run schools there is a need to introduce "Enrichment Programs" to utilize the five-month gap between SSC/HSC Examinations and completion of the admission process covering concepts of science, mathematical techniques and English communication skills. Such an enrichment program, *the Early Talent Research Participation Program*, has been, successfully, conducted by SAK since 2002 for students, who have written their 'O' Level Examinations.

CONCLUSION

Students must be taught science in such a way to develop the qualities of creative thinking and critical analysis, which are essential for a student of science [15]. Creative thinking may be developed if the students are encouraged to develop alternate explanations of the topics discussed in the class and persuaded to check the validity of their assumptions. The students can, critically, analyze a situation if they have a thorough understanding of the principles involved. Critical analysis, also, requires an awareness of the validity and the limitations of assumptions made for the solution of a problem.

In short, there are no short cuts in science. Concepts need to be developed through concrete examples using tangible objects before they can proceed to problem solving and further development of science. The examples must be selected from different areas and disciplines. Scientific and technical terms should be avoided in the beginning and must be introduced after the concepts have been mastered. More than one form of teaching aid (chalkboard, poster, 3-D model, powerpoint/flash presentation, movie, software) should be utilized. Activities, which involve students and experiments/ demonstrations, which stimulate thinking/ reasoning, are considered to be the most effective tools in concept building [9].

ACKNOWLEDGEMENTS

One of the authors (SAK) expresses his gratitude to N. M. Memon, at present Principal, Army Public School, Cantonment, Hyderabad (Sindh) for pointing out the four stages of concept building and Dr. Junaid A. Zubairi, now at Fredonia College, State University of New York for discussions of the various concepts introduced in the classroom. Last, but not the least, SAK is indebted to his wife Mrs. Khadija Ghufran Kamal for suggesting to hold on the terminologies till the very end and introduce them, when the concepts are well established in the students' minds. This work was made possible, in part, by *Dean's Research Grant* awarded by University of Karachi.

REFERENCES

[1] Ali, A., Bibi, S., Husnain, I. T., Majeed, K. (2000) Growth Dynamics of Two Sisters (one in the foster care and the other living with the natural parents), *Report*, the Aga Khan University, Karachi, Pakistan (unpublished)

[2] Ali, M., Hussain, I., Shah, S. S. (2001) Growth Dynamics, *Report*, the Aga Khan University, Karachi, Pakistan (unpublished)

[3] Bukhari, N. & Kamal, S. A. (1987) How to Cope with the Different Systems of Units? *Proc. of the National Workshop on Teaching of Physics*, Kara-chi, Pakistan, 1987, pp. 38-50

[4] Dyson, F. (1979) Time Without End: Physics and Biology in an Open Universe. *Revs. Mod. Phys.* **51**: 447-460

[5] Kamal, S. A. (1986) The Making of a Physicist. *The Karachi Physics Society Seminar*, Federal Government Urdu Science College, Karachi, Pakistan, December 25, 1986 – New undergraduate and graduate curricula were presented to suit the needs of Pakistani institutions

[6] Kamal, S. A. (1997*a*) What is Science? *The "News" International*, Karachi, July 21, 1997 – Technotalk Page

[7] Kamal, S. A. (1997*b*) What is a Scientific Method after All? *The "News" International*, Karachi, December 8, 1997 – Technotalk Page

[8] Kamal, S. A. (1998) How to Motivate the Students to Learn? Seminar, *the Aga Khan University*, Kara-chi, Pakistan, January 21, 1998

[9] Kamal, S. A. (2003). The Training of a Physicist: From Concept Building to Problem-Solving Skills. Concluding Talk, *the Second International Workshop on Physics Teaching*, Karachi, February 7-9, 2003

[10] Kamal, S. A. (2004). Mathematics Research and Teaching in University of Karachi: Challenges and Opportunities. *Winter Conference in Mathematics*, Lahore University of Management Sciences (LUMS), Lahore (Pakistan), December 3-4, 2004, abstract # 24, p 18

[11] Kamal, S. A. (2005). Mathematics Research, Teaching and Community Outreach in University of Karachi: Challenges and Opportunities. *Informative Mathematical Event*, Karachi Mathematical Association and Department of Mathematics, University of Karachi, Karachi (Pakistan), April 30, 2005

[12] Kamal, S. A. (2007). Observations and Experiments in Science, *Agahi* (Magazine of the Shaheed-é-Millat Government Degree College for Women, Azizabad, Karachi) — Intellectual Preview, pp 1-3

[13] Kamal, S. A. (2008). From Mathematics to Technology: A Bridge through Physics and Engineering, *Proc. Int. Conf. Phy. World of Today*, Department of Physics, University of Karachi, Karachi (Pakistan), pp 32-39 (invited paper)

[14] Kamal, S. A. (2009). Mathematics of Experimentation, *Nat.. Conf. Phy. World of Today*, National Center for Physics and Department of Physics, University of Karachi, Karachi (Pakistan), pp 9-10 (Prof. Dr. M. Rafi & Prof. Dr. M. Razi Hussain memorial lecture)

[15] Kamal, S. A. & Siddiqui, K. A. (1986). How to Develop Creative Thinking and Critical Analysis? *Proc. of the Second Workshop on Teaching of Physics*, Karachi, Pakistan, 1986, pp. 51-56

[16] Kamal, S. A. & Siddiqui, K. A. (1989). Basic Requirements to Train a Physicist. *Physics Education (India)* **6**: 53-61

[17] Kinchin, I. M., Hay, D. B. & Adams, A. (2000). How a Qualitative Approach to Concept-Map Analysis Can be Used to Aid Learning by Illustrating Patterns of Conceptual Development. *Educational Research* **42**: 43-57

[18] Lawless, C., Smee, P. & O'Shea, T. (1998). Using Concept Sorting and Concept Mapping in Business and Public Administration, and in Education: an Overview. *Educational Research* **40**: 219-235

[19] Siddiqui, K. A. & Kamal, S. A. (1986). Physics Makes the Deaf and the Dumb Equations of Mathematics to Speak. *Proc. of the Second Workshop on Teaching of Physics*, Karachi, p. 51

[20] Siddiqui K. A. & Kamal, S. A. (1987). A Survey of School and Pre-University Physics Education in Pakistan. In: Physics Education in Asia (Proc. of the Regional Physics Education Symposium on School-University Interface and ASPEN General Conference), ed. by Aidid, S. B., Ismail, M. Z. & Singh, M. M., Malaysian Scientific Association, pp. 81-89

[21] Summers, M. (1995). A Survey of British Primary School Teachers' Understanding of the Earth's Place in the Universe. *Educational Research* **37**: 3-19

[22] Toh, K. -A., Smee, H. -K. & Woon, T. -L. (1999). Students' Perspectives in Understanding Light and Vision. *Educational Research* **41**: 155-162

[23] Weinberg, S. (1967) A Model of Leptons. *Phy. Rev. Lett.* **13**: 168-169

[24] Zeilik, M. (1993) *Conceptual Astronomy: A Journey of Ideas* (New York: John Wiley), p. 22