A Technical Note on Bounds of Travelling Salesman Problem

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Abstract: Purpose of this paper is to highlight, an algorithm which is provided by the Cristo Nicos in (1972) is an incorrect algorithm for finding the lower bound for TSP, here we are discussing the mistake of the algorithm and also calculating the best possible value of lower bound of the problem mentioned in (Ctisto 1972), by using the same algorithm but this value could not be calculated by the author.

INTRODUCTION

Much effort has been made to find the effective lower bound for TSP, which means finding lower bound, is clearly attractive for symmetric as well as asymmetric TSP problems. In most of the lower bound algorithms the lower bound is calculated by help of assignment algorithm see [1-3] and there are some other methods see [4 and 5]. In [3] a method is proposed to find the lower bound and declared to be a lower bound near to optimal, the author professed that the calculated value 214 is a lower bound for the optimal. He had also asserted (without any proof) that the optimal TSP length is 216. So, according to this assumption of optimal length, his calculated lower bound seems to be correct, but infect his assumption about the optimal TSP length is actually wrong because there exist a TSP path (of the proposed problem) having the length 212 hence obviously it contradicts with the result of suggested lower bound by the author because it is less than the lower bound calculated by the author. In this paper we are providing the actual lower bound for the proposed example by using the algorithm given by the author also detecting the mistake of the algorithm which proves that it is an is incorrect algorithm.

1. The steps of the algorithm presented by Nicos are as follows.

Step # 1: Set a matrix M equal to the initial distance matrix d_{ij} and set L=0.

Step # 2: If the matrix satisfies the triangularity condition of metric space, go to step 3; if not, compress M until $m_{ij} \leq m_{ik} + m_{kj}$ for any value of k.

Step # 3: Solve the assignment problem by using matrix M and let V(AP) be the value of this solution. Set L =L+ V(AP).

Step # 4: Contract the matrix M by replacing sub tours (formed as a result of the solution to the assignment problem at step 3) by single nodes by using equations,

$$d_1(S_{1,i}, S_{1,j}) = \min_{k_i \in S_{1,i},\, k_j \in S_{1,j}} [f_1(k_i, k_j)],$$

$$d_2(S_{2,i}, S_{2,j}) = \min_{k_i \in S_{2,i},\, k_j \in S_{2,j}} [f_2(k_i, k_j)],$$

Step # 5: If the contracted matrix M is 1 by 1 matrix go to step 6 otherwise return to step 2.

Step # 6: The value of L is a lower bound to the value of the TSP.

SOLUTION OF THE EXAMPLE

Consider the weighted matrix,

	1	2	3	4	5	6	7	8	9	10
1	X	32	41	22	20	57	54	32	22	45
2	32	X	22	30	42	51	61	20	54	31
3	41	22	X	63	41	30	45	10	60	36
4	22	30	63	X	36	78	72	54	20	64
5	20	42	41	36	X	45	36	32	22	28
6	57	51	30	78	45	X	22	32	67	20
7	54	61	45	72	36	22	X	41	57	10
8	32	20	10	54	32	32	41	X	50	32
9	22	54	60	20	22	67	57	50	X	50
10	45	31	36	64	28	20	10	32	50	X

The result of the first assignment that is value of AP_o is 184, and the resulting cycles are (1,5), (2,8,3),(6,7,10) and (4,9) and author has consider the indices of these cycles as S_{11},S_{12},S_{13} and S_{14} respectively. According to the step 4, after the contraction and second assignment the value is $V(A_1) =$ 20 and the value of the lower bound will become 204, Again by performing the step of contraction and the third assignment, the value is $V(A_2) = 10$ which makes the value of lower bound L= 204+10 = 214. But there is a TSP path whose value is less than the lower bound calculated by using the above procedure. The optimal TSP path of the proposed example is,

1→5→10→7→6→3→8→2→4→9→1.

with the value 212.

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Actually the author did not mention in his paper that the cycles can be taken in different combination, after the first assignment. According to him these cycles can be selected at random but in this paper we are showing that by taking the different combination of cycles according to their indices we can have the different values in which some of the them are lower bound and some are not, which makes the author's algorithm incorrect, because sometimes the calculated value is greater than the optimal value for TSP as we have mentioned earlier.

In actual it is not cleared by the author that after the first assignment what could be the best possible combination of cycles in respect to their indices, as far as the proposed example is concerned the author has randomly selected the indices of the cycles as he has mentioned in his paper. Since he did not declare any criteria of taking the indices of the cycles, but infect changing the indices of the cycles may yield different contracted form. And definitely the further calculation will be changed in these sequences. Here we are providing the arrangement of cycles by which we will have the lower bound for this particular proposed example.

Since the given matrix is already in a compressed form so, according to step **3** after the first assignment the cycles are,

(1,5), (4,9), (2,8,3), (6,7,10)

Consider the cycles (6,7,10) as S_{11} and consider (2,8,3) as S_{12} and by considering the cycle (4,9) as S_{13} and also consider the cycle (1,5) as S_{14} .

By using these above mention indices of cycles perform the step of contraction according to the formula which is given by the author for the contraction in step 4, the contracted form of the matrix will be.

After first contraction the matrix will be,

	1	2	3	4
1	∞	8	30	8
2	8	∞	18	0
3	42	30	∞	2
4	20	12	2	∞

After the second compression the matrix will be,

	1	2	3	4
1	∞	8	10	8
2	8	∞	2	0
3	22	14	∞	2
4	10	12	2	∞

According to the algorithm after the second assignment the new form of the matrix will become,

	1	2	3	4
1	∞	0	2	0
2	0	∞	2	0
3	12	12	∞	0
4	0	10	0	∞

The value of the second assignment $V(A_1) = 20$, and the new lower bound will become

$L= V(A_0)+V(A_1)$ which is equal to L= 184+20 =204.

At this stage the value of the second assignment is equal to the value which is calculated by the author with his combination of cycles, but when we perform the next step of contraction the new matrix will be totally changed which is,

	1	2
1	∞	0
2	0	∞

And by using the above matrix the value of the third assignment will be $V(A_2) = 0$, and according to the formula the lower bound will become L=204, As optimal value of this particular problem is 212, which means the calculated lower bound is not so close to the optimal value.

In our opinion the author's algorithm is an incorrect because of the fact that there are no hard and fast criteria about assigning the indices of the cycles so the obvious criteria would be on trial bases.

CONCLUSION

In this paper we have proved that the author has proposed an incorrect algorithm to find the lower bound of TSP, as it provides the different values for different combination of cycles of which some of them are not even lower bounds.

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