2-Approximate Algorithm for Travelling Salesman Problem with the Insertion of Cycles

Nasiruddin Khan[1], Fozia Hanif Khan[2,*] and Syed Inayatullah[3]

[1]*Department of Mathematics University of Karachi,* [2]*Fazaia Degree College Faisal, Department of Mathematics and* [3]*Department of Mathematics, University of Karachi*

Abstract: This paper proposes a 2- approximate algorithm for Travelling salesman problem by using the neighborhood search and the insertion of cycles to find the near optimal tour. This algorithm describes the two faceted decision structure related to the problem. First finding the cycle and searching the neighborhood, second, insertion of cycles according to the neighborhood, both symmetric and asymmetric problems can be solved by using this algorithm. The process gradually builds a route by inserting a cycle at each step in the neighborhood of the forthcoming vertex and performing a local optimization. This is done while checking the feasibility of the remaining part of the route.

Key Words: Cyclic insertion, Triangular inequality, Travelling salesman problem.

INTRODUCTION

Traveling salesman problem has been used as (TSP) in the literature is one of the most studied problems in the combinational optimization. The problem can be stated in the following terms: given n cities and a matrix of distance $D = (d_{ij})$, where d_{ij} is the distance from the city i to city j, starting at any arbitrary city, visit each city exactly once and return to the originating city in a way that minimizes the total distance traveled. The TSP can be seen as a graph theory problem if the cities are identified with the nodes of a graph, and the link between the cities is associated with the arcs. A weight corresponding to the intercity distance is assigned to each edge. In this way we can say TSP is equivalent to find the minimal weighted Hamiltonian circuit in the graph.

According to [1], the classification of TSPs depends upon the properties of the distance matrix D, if $d_{ij} = d_{ji}$ for every pair in $N = \{1,2\ldots\ldots n\}$, then TSP is termed as symmetric other wise asymmetric. If $d_{ij} \leq d_{ik} + d_{kj}$ for all i, j and k in N, then the TSP satisfies the triangle inequality.

Because of the difficulty in solving TSP optimally (it has been shown by [2],[3] and [4] to be NP complete) several approaches have been proposed obtaining near optimal solution in a reasonable time. Most of which satisfy the triangle inequality.

In this paper we proposed a generalized insertion of cycle's for TSP, here we are searching for the neighborhood to find the near optimal tour, considering the tour improvement heuristic by taking a complete tour and repeatedly improving it. At each insertion a number of possible changes are considered, and the best change found is actually made. This process is continuous until no change considered produces an improvement. The most common tour improvement techniques are 2-opt [5] and 3- opt [6]. This problem has received the attention before, many numerous approximate algorithms have been proposed that find relatively good solutions quickly.

Most of these algorithms fall in to two categories, [7] tour construction, and the tour improvement. A typical tour construction algorithm starts with a subset of points linked in a cycle and adds the other one by one until the tour is complete. Many variations have been described in [8-13]. Application of TSP arises in machine scheduling when n jobs are assigned with relative time, their economic importance is marked by their presence in manufacturing and service industries.

This paper is organized as follows. Section 1 gives the introduction, section 2 gives the notations for the insertion of cycles and short circuiting via triangular inequality for the problem, section 3 develops the computational sufficient algorithm to find the near optimal tour also discusses the implementation of algorithm on the optimal salesman problem, section 4 gives the 2-approximate prove of the proposed algorithm, section 5 shows the example of the proposed algorithm, section 6 analyzes the conclusion and section 7 gives the references.

NEIGHBORHOOD OF A NODE

Neighborhood for any node i for a tour is defined as a set of nodes (belonging to the same tour) that are closest to it. In our algorithm each closest node is obtained according to the reduced weighted matrix, the nearest node is the unvisited node whose distance is zero or least as compare to other unvisited node.

*Address correspondence to this author at the PVC of Karachi University; E-mail: drkhan.prof@yahoo.com

0250-5363/07

Tour Construction

Starting with any arbitrary node say i_1 build a sequence i_1, i_2, i_3 ... i_n, i_1 by successively including nearest neighborhood nodes into the sequence. For example, from i_1 proceed to the nearest node say i_2, from i_2 move to the nearest node not yet visited, and so on up to last unvisited node say i_n , and then from i_n directly join the initial node i_1.

Clearly this method of tour construction is a good initial approximation to the optimal tour and also there are many variations for example see [14], [15], and [16] of the above nearest- neighbor rule.

Tour Extension via Cyclic Insertion

Find all minimal cycle for each vertex except from the starting point if possible from the reduced weighted matrix. Some of the cycles may be of equivalent length.

In the neighborhood of starting point replace the associated minimal cycle that is node i is replaced by its minimal cycles $i \rightarrow k \rightarrow i$. Before insertion we will also cross out all equivalent cycles from the set of minimal cycle.

2.3. Short Circuiting via Triangular Inequality:

Triangular inequality guaranties that the short-circuiting can only shorten the length of the tour [17].

According to the triangular inequality we can replace d_{ij} by $d_{ik} + d_{kj}$ when $d_{ik} + d_{kj}$ is the shortest path from i to j.

$$d_{ij} > d_{ik} + d_{kj}$$

In this way we can short-circuit any vertex or edge from the tour that includes the segment…..$i \rightarrow k \rightarrow j$…. and k already appears on the tour, so we can remove k and connect i directly to j, but

$$d_{ij} < d_{ik} + d_{kj} .$$

Tour Improvement

After short-circuiting via triangular inequality there will be a tour which is not supposed to be optimal so apply the tour improvement step by trying to reinsert each vertex (except from the starting point) in between all two consecutive vertices of the tour in order to minimize the tour's length. If there is some improvement in the tour length then move the vertex to new position otherwise the vertex will remain at the previous position the procedure stops when there is no change in the current tour.

Different procedure of tour improvement has been used for example [18], [19], [20], [21], and [22].

Steps of Algorithm

Step # 1: Select any vertex as a starting point and construct the tour.

Step # 2: Apply tour extension.

Step # 3: Short-circuiting the tour.

Step # 4: Apply the tour improvement step.

Implementation of the Algorithm on OSP

Like all algorithm for TSP this algorithm can also be implemented on optimal salesman problem OSP that is TSP for repeating vertex by considering the shortest distances matrix, for OSP we will continue with the same procedure as for TSP in the end we replace the tour by its original distance using original weighted matrix. For OSP the final tour will be with repeating vertices. It is obvious that the value of the optimal tour for OSP should be equal or less than the TSP. since we are using the shortest distance matrix that is the triangle inequality of the given matrix.

4. THE ABOVE ALGORITHM IS 2-APPROXIMATE ALGORITHM FOR TSP

Theorem: There is a 2-approximate algorithm for TSP.

Lemma 1: $S^* \leq$ TSP

Lemma 2: $C' \leq$ TSP

Proof:

Let T be the tour obtained by the above algorithm.

Let C be the tour after the insertion of cycles in the constructed tour.

And C' is the length of the cycles.

And N is the length of the non zero edges.

Let $S^* = C - C' - N$

$S^* \leq$ TSP (From lemma 1)

Because S* contains only zero edges not the complete tour.

$C' \leq$ TSP (From lemma 2)

Since inserting the minimal cycles for each vertex only once if exist.

So we can say,

$T \leq S^* + C' + N$ = Total length of the initial tour since T is found by triangle inequality.

But it is clear that,

$T \leq S^* + C'$ as well,

$T \leq TSP + TSP$

Therefore,

$T \leq 2TSP$.

EXAMPLE

Consider the weighted matrix,

	A	B	C	D	E
A	∞	3	6	2	3
B	3	∞	5	2	3
C	6	5	∞	6	4
D	2	2	6	∞	6
E	3	3	4	6	∞

The reduced weighted matrix,

	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

By using the reduced weighted matrix the constructed tour will be A→D→B→E→C→A and the set of cycles are B→D→B, D→ B→D, C→E→C, E→C→E. After the insertion of cycles the extended tour will be A→D→B→D→B→E→C→E→C→A as shown in Fig. (**1**). Finally after the short-circuiting and the tour improvement the near optimal tour will be

A→D→B→C→E→A

And the total length of the tour is 16 which is also coincidently optimal.

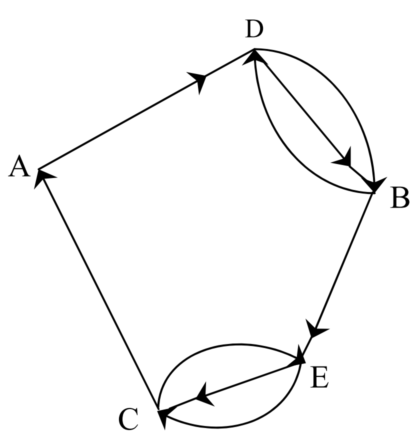

Fig. (1).

Consider another example of the proposed algorithm,

	A	B	C	D	E
A	∞	7	12	5	6
B	7	∞	8	11	10
C	12	8	∞	8	6
D	5	11	8	∞	3
E	6	10	6	3	∞

The reduced weighted matrix,

	A	B	C	D	E
A	∞	0	6	0	1
B	0	∞	0	4	3
C	6	0	∞	2	0
D	2	6	4	∞	0
E	3	5	2	0	∞

By using the reduced weighted matrix the constructed tour by using the nearest-neighbor rule will be A→B→C→E→D→A and the set of cycles are B→C→B, C→ B→C, D→E→D, E→D→E. After the insertion of cycles the extended tour will be A→B→C→B→C→E→ D→E→D→A as shown in Fig. (**1**). Finally after the short-circuiting and the tour improvement the near optimal tour will be

A→B→C→E→D→A

And the total length of the tour is 29 which is also coincidently optimal.

Consider the second constructed tour from the same reduced weighted matrix,

A→D→E→C→B→A, after the insertion of cycles the extended tour will be A→D→E→D→E→C→B→C→B→, finally after short-circuiting and the tour improvement, final tour will be

A→D→E→C→B→A, which is also optimal with cost 29.

CONCLUSION

In this paper we have presented a 2- approximate algorithm for TSP and hence for OSP by considering the approach of insertion of cycles according to the neighborhood of the successive vertex in the constructed tour. According to the algorithm first compute the tour by considering the zero vertices of the reduced matrix, to avoid traversing any vertex more than once directly go to the next non zero vertex to complete the tour. Find the cycles from the reduced weighted matrix and insert the minimal cycles for the vertex only ones. Short-circuit the vertices and the edges that are encountered more than once by using the triangle inequality to get the tour. In this way we have a tour with the appearance of each vertex only once. In the end we have applied the tour improvement step for of each vertex (except the starting point) in all the edges one by one until there is no change in the current path and that would be considered as a near optimal solution of the given graph.

REFERENCES

[1] JEFFREY L. ARTHUR AND JAMES O. FRENDEWEY, Generating Travelling salesman problem with known optimal. Oregon State University, (1998).

[2] R. M. KARP, Reducibility among combinational problem. In Complexity of computer Computation 9R. E. Millerand J. W. Thatcher, Eds, (1972).

[3] R. M. KARP, On the complexity of Combinational problems. Networks.,(1975), 45-68.

[4] C. H. PAPADIMITRIOUS, The Euclidian traveling salesman problem is NP complete., (1977), 5, 45-68.

[5] CROSE, A., A Method for Solving Travelling salesman problem. Opns. Res., (1958), 16,791-812.

[6] LIN, S, Computer Solutions of traveling salesman problem. Bell Syst. Tech. J., (1965), 44, 2245-2269.

[7] GEOFFREY ZWEIG, An effective tour construction procedure for the traveling salesman problem. International Computer Science Institute, Berkeley, California., (1995).

[8] JOHNSON, D. S., Local optimization and the Travelling Salesman problem. In Lecture notes in computer Science 443, M. S. Paterson (ed). Springer-Verlag, Berlin., (1990), 446-461.

[9] GOLDEN, B., L. BODIN, T. Doyle And W. R. Stewart Jr., Approximate travelling salesman algorithm. Opns. Res., (1980), 28, 695-711.

[10] LIN, S., AND B. W. KERNIGHAN, An effective Heuristic algorithm for the traveling salesman problem. Opns. Res., (1973), 21, 498-516.

[11] GOLDEN, B. L., AND W. R. STWERT Jr., Empirical analysis of heuristics. In the traveling Salesman problem. A guided Tour of Combinational optimization, E. L. Lawler, J. K., (1985).

[12] GENDREAU, M., A. HERTHZ AND G. LAPORTE, New insertion and post optimization Procedure for traveling salesman problem. Opns. Res., (1992), 40, 1086-1094.

[13] BENTLEY, J. L., Fast algorithm for Geometric Travelling Salesman problem. ORSA J. Cambridge Philos. Soc., (1992), 55,299-327.

[14] DACEY, M. F., Selection of an initial solution for traveling salesman problem. Opns. Res., (1960), 8, 133-134.

[15] KARG, L. AND R. M. KARP, A Heuristic Approach to solving Traveling salesman problem, Management Sci., (1964), 10,225-248.

[16] STOUDHAMMER, J. AND M. ASH, A Sufficiency Solution of Traveling Salesman Problem, Sys. Aer. Corp., Santa Monica, California., (1966).

[17] JONATHAN HUI AND BONNIE KIRKPATRICK, Combinational Algorithms and data structure., (2005), CS207.

[18] BARACHET, L. L., Graphic Solution of Traveling Salesman Problem, Opns. Res., (1957), 5, 841-845.

[19] CROES, G. A., A Method of Solving Traveling Salesman Problem, Opns. Res., (1958), 6,791-812.

[20] LIN, S., Computer Solution of the traveling salesman problem, Bell System Tech.J., (1965), 44, 2245-2269.

[21] REITER, S. AND G. SHERMAN, Discreet Optimization SIAM., (1965), 13, 864-889.

[22] ROBERTS. S. M. AND B. FLORES, An Engineering Approach to the Traveling Salesman Problem, Management Sciences., (1966), 13, 269-288.